The equation of state in QCD at finite chemical potential from lattice simulations

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What we want to study?

How to study it

Result
What we want to study:

The Quark-Gluon-Plasma and the transition to hadrons
The \((T, \mu_B)\)-phase diagram of QCD

Our observables:
Last Year: \(T_c\)
This year: \(P, \epsilon, S, N_B\) and \(I\) along trajectories of constant \(\frac{S}{N_B}\) and their Taylor coefficients
Ways to study the QGP:

- Time travel ✗
  The universe was not transparent for $\gamma$ yet
- Looking at particle collision
  Yes, heavy ion collisions; but I'm a theorist
- Solving the theory of the strong interaction: QCD
  Yes, via Lattice QCD
Why LQCD?

\[ L_{QCD} = -\frac{1}{4} F_{\mu\nu}^a F^{a,\mu\nu} + \bar{\psi} (i\gamma_\mu D^\mu - m) \psi \]

- Because of the strong coupling and the self interaction of gluons perturbation theory is not feasible
- Path integral quantisation:

\[
\langle 0 | T \hat{\phi}_1 \ldots \hat{\phi}_n | 0 \rangle = \frac{\int D\phi \hat{\phi}_1 \ldots \hat{\phi}_n e^{i \int dx \ L}}{\int D\phi \ e^{i \int dx \ L}}
\]

First problem: There are many points in space-time \( D\phi = \prod_i d\phi(x_i) \)
Solution: Replace continuous space by a discrete 4d lattice
How do we do that?

- We look at a 4d space-time lattice with size $N_s^3 \times N_t$
- Fermions live on the lattice sites gauge fields live on the links
- We use Euclidean space-time: $t \rightarrow i\tau$
- We can do Monte-Carlo-Simulations (with importance sampling) to solve our integrals
- Everything is determined in terms of our lattice spacing $a$
- $a$ has to be determined by comparison with physical observables (for example $a = \frac{(am_p)_{\text{lattice}}}{938 \text{ MeV}}$)
- We have to take the limits $a \rightarrow 0$, $N_s \rightarrow \infty$
- To do thermodynamics: $T = \frac{1}{aN_t}$
Why aren’t we finished yet?

- Simulations take a lot of computer time
- Not everything can be calculated directly. For example:
  - Only observables that can be calculated in Euclidean space
  - Only thermal equilibrium
  - Only simulations at \( \mu_B = 0 \Rightarrow \langle n_B \rangle = 0 \)

heavy ion collision experiments

Solution: Analytical continuation

\[
\frac{\partial}{\partial \mu} \left( \frac{p}{T^4} \right)_{T_c(\mu)}
\]
Analytic continuation

\[ \frac{d(p/T^4)}{d\mu} \]

\[ T_{c(\mu)} \]

Roberge-Weiss

real chemical potentials

lattice simulations

\[ \mu^2/T^2 \]

continuation

\[ \frac{d(p/T^4)}{d\mu} \]

\[ T_{c(\mu)} \]
The Analysis

1. Do the simulations
2. Make a fit in the $T$ direction
3. Determine everything you need for the observables
4. Make a fit in the $\mu_B$ direction
5. Make a fit in the $\frac{1}{N_t^2}$ direction ($a \rightarrow 0$ extrapolation)
6. Determine the observables
Error estimation

- Statistical error:
  Bootstrap method

- Systematic error:
  Using different way of analysis, combining them in a histogram:
  - 2 fit functions for the $T$ direction
  - 4 fit functions in the $\mu_B$ direction
  - Doing continuum extrapolation and $\mu_B$-fit in one or two steps
  - 2 ways of measuring $a$

This adds up to 64 ways of analysis
$T_c$
The Taylor coefficients of \( \frac{P}{T^4} = c_0 + c_2 \left( \frac{\mu_B}{T} \right)^2 + c_4 \left( \frac{\mu_B}{T} \right)^4 + c_6 \left( \frac{\mu_B}{T} \right)^6 \)
Trajectories

Budapest-Wuppertal preliminary
Equation of state
Conclusion

- Lattice QCD is a non perturbative method to study QCD
- Results a finite $\mu_B$ can be obtained by analytical continuation
- The critical temperature can be extrapolated up to 400 MeV
- Results for the Taylor coefficients of $P/T^4$
- $S/N_B$ trajectories to match heavy ion beam energies at RHIC
- Equation of state at different $S/N_B$ values