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THE NEW PHYSICS FRONTIERS IN THE LHC- 2 ERA
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THE SOFT-WALL MODEL


\[ S = \int d^4x dz \sqrt{g} e^{-\Phi(z)} \mathcal{L}(x, z), \]  

(1)

\[ g = |\det g_{MN}| (M, N = 0, 1, 2, 3, 5) \]

dilaton field \( \Phi(z) = k^2 z^2 \)

\[ \eta_{\mu\nu} = \text{diag}(1, -1, -1, -1). \]

\[ 0 \leq z < \infty. \]

The metric of AdS space is given in Poincare coordinates and its radius has been set \( R = 1 \):

\[ ds^2 = \frac{1}{z^2} \left( -dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu \right), \quad \mu, \nu = 0, 1, 2, 3. \]
\(\rho\)-mesons in soft-wall model


Let us introduce in the bulk of AdS space two gauge fields \(A^M_L\) and \(A^M_R\), which transform as a left and right chiral fields under \(SU(2)_L \times SU(2)_R\) chiral symmetry group of the model. The chiral symmetry group is broken to the isospin group \(SU(2)_V\) (in vector representation) due to the interaction of the bulk gauge fields with the scalar field \(X\). According to the AdS/CFT correspondence the bulk \(SU(2)_V\) symmetry group is the symmetry group of the dual boundary theory, i.e. isospin symmetry of QCD and the \(\rho\) meson triplet is described by this representation of the \(SU(2)_V\) group. From the gauge fields \(A^M_L\) and \(A^M_R\) one can construct a bulk vector field \(V^M = \frac{1}{\sqrt{2}} (A^M_L + A^M_R)\) and the axial vector field \(A^M = \frac{1}{\sqrt{2}} (A^M_L - A^M_R)\). According to the AdS/CFT correspondence of vector field the UV boundary value of the Kaluza-Klein modes of bulk vector field correspond to the vector meson series of the dual theory. Since the \(\rho\) meson is the lightest vector meson in particle physics it corresponds to the first state of these modes. Action for the gauge field sector will be written in terms of bulk vector and axial-vector fields as following:

\[
S_{\text{gauge}} = -\frac{1}{4g_5^2} \int d^5x \sqrt{g} e^{-\Phi(z)} Tr \left[ F^2_L + F^2_R \right] = -\frac{1}{4g_5^2} \int d^5x \sqrt{g} e^{-\Phi(z)} Tr \left[ F^2_V + F^2_A \right],
\]

where the field stress tensor is \(F_{MN} = \partial_M V_N - \partial_N V_M - i[V_M, V_N], V_M = V^a_M t^a, t^a = \sigma^a/2\).
Equation of motion for Fourier components $\tilde{V}_\mu^a(p, z)$ is easily obtained from the action (4) and has the form

$$\partial_z \left[ \frac{1}{z} e^{-k^2 z^2} \partial_z \tilde{V}_\mu^a(p, z) \right] + p^2 \frac{1}{z} e^{-k^2 z^2} \tilde{V}_\mu^a(p, z) = 0. \quad (5)$$

The $\tilde{V}_\mu^a(p, z)$ can be written as $\tilde{V}_\mu^a(p, z) = V_\mu^a(p)V(p, z)$ and at UV boundary $V(p, z)$ satisfies the condition $V(p, \epsilon) = 1$, since it is assumed that 4-dimensional physical world resides on this boundary of the AdS space.

Equation (5) is reduced to the Schroedinger equation form and has a solution in terms of Laguerre polynomials $L_m^n$:

$$\psi_n(z) = e^{-k^2 z^2/2} (kz)^{m+1/2} \sqrt{\frac{2n!}{(m+n)!}} L_m^n (k^2 z^2). \quad (6)$$

For the eigenvalues $m_n^2$ there is a linear dependence on the number $n$: $m_n^2 = 4k^2(n + 1)$, which enables us to fix the free parameter $k$. In the AdS/CFT correspondence $m_n^2$ is identified with the mass spectrum of the vector mesons in the dual boundary QCD. For the $a$-meson we have $m = 1$ and the $V_n(z)$ becomes

$$V_n(z) = k^2 z^2 \sqrt{\frac{2}{n+1}} L_n^1 (k^2 z^2). \quad (7)$$
Nucleons in soft-wall model


It should be noticed that a nucleon doublet in hard-wall model is described by a pair of bulk fermions, because the left- and right-handed components of the nucleon operator on the boundary are described by the two bulk fermion fields having opposite signs of 5-dimensional mass $M$ (the detailed substantiation can be found in [10]). However, the soft-wall model Lagrangian contains an additional term $\Phi \bar{\Psi} \Psi$, which describe the coupling of dilaton field with the bulk fermion fields ([6]) and the sign of this term for the second fermion field is chosen an opposite to the one for the first fermion [12]. So, in order to describe the nucleon doublet in the boundary QCD it is necessary to introduce two bulk fermions $N_1$ and $N_2$ having opposite signs of $M$ and $\Phi \bar{\Psi} \Psi$ term, then eliminate the extra chiral components on the UV boundary by the boundary conditions. Let us demonstrate the profile function derivation for the fermion field $\Psi_1(x, z)$ in the bulk of AdS space (2). The action for this field without taking into account the interaction with the gauge fields, can be written as following:

$$S_{F_1} = \int d^4x dz \sqrt{g} e^{-\Phi(z)} \left[ \frac{i}{2} \bar{\Psi}_1 e_A^N \Gamma^A D_N \bar{\Psi}_1 - \frac{i}{2} (D_N \Psi_1)^\dagger \Gamma^0 e_A^N \Gamma^A \Psi_1 - (M + \Phi(z)) \bar{\Psi}_1 \Psi_1 \right],$$

(8)

where $e_A^N = z \delta_A^N$ is the inverse vielbein and the covariant derivative is $D_N = \partial_N + \frac{1}{8} \omega_{NAB} [\Gamma^A, \Gamma^B]$. Non-zero components of spin connection are: $\omega_{\mu z \nu} = -\omega_{\mu z \nu} = \frac{1}{2} \eta_{\mu \nu}$. The 5-dimensional gamma matrices obey the anticommutation relation $\{\Gamma^A, \Gamma^B\} = 2\eta^{AB}$ and are defined as $\Gamma^A = (\gamma^\mu, -i\gamma^5)$. 
Equation of motion obtained from the action (14) has a form

$$\left[ ie^N_A \Gamma^A D_N - \frac{i}{2} (\partial_N \Phi) e^N_A \Gamma^A - (M + \Phi(z)) \right] \Psi_1 = 0. \quad (9)$$

The $n$-th normalized Kaluza-Klein mode $f_{L,R}^{(n)}(z)$ of the solutions $f_{L,R}$ with $p^2 = m_n^2$ can be expressed in terms of Laguerre polynomials:

$$f_{1L}^{(n)}(z) = n_{1L} (kz)^{\alpha} L_n^{(\alpha)}(kz),$$
$$f_{1R}^{(n)}(z) = n_{1R} (kz)^{\alpha^{-\frac{3}{2}}} L_n^{(\alpha-1)}(kz).$$

The parameter $\alpha$ is related with the 5-dimensional mass $M$ via $\alpha = M + \frac{1}{2}$. It relates the mass of the $n$-th mode $m_n$ with the number $n$ in the following $m_n^2 = 4k^2(n + \alpha)$, which serves as another condition to fix the parameter $k$ of the model. The constants $n_{L,R}$ are found from the normalization condition

$$n_{1L} = \frac{1}{k^{\alpha^{-1}}} \sqrt{\frac{2\Gamma(n+1)}{\Gamma(\alpha+n+1)}},$$
$$n_{1R} = n_{1L} \sqrt{\alpha + n}.$$

$$f_{1L} = f_{2R}, \quad f_{1R} = -f_{2L},$$
BULK INTERACTION AND THE $g_{\rho NN}$ COUPLING CONSTANT

$$S_{\text{int}} = \int d^4 x dz e^{-\Phi(z)} \sqrt{g} \mathcal{L}_{\text{int}}.$$

$$\langle J_\mu \rangle = -i \frac{\delta Z_{\text{QCD}}}{\delta V_\mu} \bigr|_{V_\mu = 0} \quad Z_{\text{QCD}} = e^{iS_{\text{int}}},$$

First, $\mathcal{L}_{\text{int}}$ contains a term of minimal gauge interaction of the vector field with the current of fermions

$$\mathcal{L}^{(0)}_{\rho NN} = \bar{N}_1 e_A^M \Gamma^A V_M N_1 + \bar{N}_2 e_A^M \Gamma^A V_M N_2,$$

$$g^{(0)nm}_{\rho NN} = \int_0^\infty \frac{dz}{z^4} e^{-\Phi(z)} V_0(z) \left( f_{1L}^{(n)*}(z) f_{1L}^{(m)}(z) + f_{2L}^{(n)*}(z) f_{2L}^{(m)}(z) \right).$$

$$L_{FNN} = i k_1 e_A^M e_B^N \left[ \bar{N}_1 \Gamma^{AB} (F_L)_{MN} N_1 - \bar{N}_2 \Gamma^{AB} (F_R)_{MN} N_2 \right]$$

$$+ \frac{i}{2} k_2 e_A^M e_B^N \left[ \bar{N}_1 X \Gamma^{AB} (F_R)_{MN} N_2 + \bar{N}_2 X^+ \Gamma^{AB} (F_L)_{MN} N_1 - \text{h.c.} \right].$$

$$g^{(1)nm}_{\rho NN} = -2 \int_0^\infty \frac{dz}{z^3} e^{-k^2 z^2} V_0'(z) \left[ k_1 \left( f_{1L}^{(n)*} f_{1L}^{(m)} - f_{2L}^{(n)*} f_{2L}^{(m)} \right) + k_2 v(z) \left( f_{1L}^{(n)*} f_{2L}^{(m)} + f_{2L}^{(n)*} f_{1L}^{(m)} \right) \right]$$

$$f_{\rho}^{nm} = 4 m_N \int_0^\infty \frac{dz}{z^3} e^{-k^2 z^2} V_0(z) \left[ k_1 \left( f_{1L}^{(n)*} f_{1R}^{(m)} - f_{2L}^{(n)*} f_{2R}^{(m)} \right) + k_2 v(z) \left( f_{1L}^{(n)*} f_{2R}^{(m)} + f_{2L}^{(n)*} f_{1R}^{(m)} \right) \right].$$
$g_{\rho NN}$ constant as the sum of two terms, the first of which is $g_{\rho NN}^{s.w.} = g_{\rho NN}^{(0)nm} + g_{\rho NN}^{(1)nm}$

**NUMERICAL ANALYSIS**

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**TABLE I**: Numerical results for $k = 0.389$, $m_{\rho}^{s.w.} = 0.778$, $k_1 = -0.78$, $k_2 = 0.5$, $\sigma = (0.368)^3$ and $m_q = 0.00145$ GeV.
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**TABLE II:** Numerical results for $k = 0.389$, $m_{\rho}^{s.w.} = 0.778$, $k_1 = -0.78$, $k_2 = 0.5$, $\sigma = (0.326)^5$ and $m_q = 0.0023$ GeV.
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TABLE III: Numerical results for \( k = 0.35, m_{\rho^{s.w.}} = 0.7, k_1 = -0.78, k_2 = 0.5, \sigma = (0.368)^3 \) and \( m_q = 0.00145 \text{ GeV} \).
<table>
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TABLE IV: Numerical results for $k = 0.35$, $m_{ρ}^{s.w.} = 0.7$, $k_1 = -0.78$, $k_2 = 0.5$, $σ = (0.326)^3$ and $m_q = 0.0023$ GeV.
Thank you for attention