Phase structure of a holographic double monolayer semimetal

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Based on:

AdS/CFT correspondence

Gravity theory

Type IIB Superstring in $\text{AdS}_5 \times S^5$

classical regime

Gauge theory

$\mathcal{N} = 4 \ SU(N)$ SYM in 4-d

strong coupling
Graphene

- Graphene \(\rightarrow\) 2+1-dim semimetal

- Low energy excitations \(\rightarrow\) relativistic massless fermions interacting through electromagnetic forces

- Graphene is a strongly interacting system \(\rightarrow\) AdS/CFT correspondence
Double monolayer graphene

- **Double monolayer graphene (DMG)** → two monolayers of graphene brought into close proximity but still separated by an insulator
  - no direct transfer of electric charge carriers between the layers

- **Exciton** → bound state of fermion and antifermion
  - intra-layer
  - inter-layer

- Study the formation of these exciton condensates in DMG with an external magnetic field \((B)\) and balanced charge densities \((q)\) at both zero and finite temperature
We study the D3/probe D5-$D\bar{5}$ system

- Introduce an external magnetic field and balanced charge densities on the D5-$D\bar{5}$
Setup (at zero temperature)

- Stack of $N$ D3-branes $\rightarrow$ AdS$_5 \times S^5$ background

$$ds^2 = r^2 \left( -dt^2 + dx^2 + dy^2 + dz^2 \right)$$

$$+ \frac{1}{r^2} \left( dr^2 + r^2 d\psi^2 + r^2 \sin^2 \psi d\Omega_2^2 + r^2 \cos^2 \psi d\tilde{\Omega}_2^2 \right)$$

- Embed $N_5$ D5 and $\overline{D5}$ probes in this background ($N_5 \ll N$)

- DBI action

$$S = -T_5 N_5 \int d^6 \sigma \sqrt{-\det(g + 2\pi \ell_s^2 F)}$$

- Charge density $q$ and external magnetic field $B$

$$F = A'_0(r) dr \wedge dt + B dx \wedge dy$$

$$q \approx - \frac{\delta S}{\delta A'_0}$$
Setup

- Ansatz for the embedding of the D5/D5

\[
\begin{array}{cccccccc}
  t & x & y & z & r & \psi & \theta & \phi & \tilde{\theta} & \tilde{\phi} \\
\text{D3} & \times & \times & \times & \times & - & - & - & - & - \\
\text{D5/D5} & \times & \times & \times & z(r) & \times & \psi(r) & \times & \times & - & - \\
\end{array}
\]

- Boundary conditions

\[
\lim_{r \to \infty} \psi(r) = \frac{\pi}{2} \\
\lim_{r \to \infty} z(r) = \pm \frac{L}{2}
\]
Symmetry breaking

The geometry of the D5-\(\overline{D5}\) can break two symmetries

1. \(\psi(r) \neq \frac{\pi}{2} \rightarrow SO(3) \times SO(3) \rightarrow SO(3) \times SO(2)\)
   - intra-layer condensate \(\psi(r) \approx \frac{c}{r^2}, \quad c \propto \langle \bar{\psi}_i \psi_i \rangle\)

2. \(z(r) \neq \text{const} \rightarrow U(N_5) \times U(N_5) \rightarrow U(N_5)\)
   - partial annihilation of D5 and \(\overline{D5}\)
   - inter-layer condensate \(z(r) \approx \pm \frac{L}{2} \mp \frac{f}{r^5}, \quad f \propto \langle \bar{\psi}_1 \psi_2 \rangle\)
Types of solutions

1. \( \psi(r) = \frac{\pi}{2} \) and \( z(r) = \pm \frac{L}{2} \) → symmetric phase

2. \( \psi(r) \neq \frac{\pi}{2} \) and \( z(r) = \pm \frac{L}{2} \) → intra-layer condensate

3. \( \psi(r) = \frac{\pi}{2} \) and \( z(r) \neq \pm \frac{L}{2} \) → inter-layer condensate

4. \( \psi(r) \neq \frac{\pi}{2} \) and \( z(r) \neq \pm \frac{L}{2} \) → intra/inter-layer condensates
Free energy

Which configuration is favored?

- Compare the free energies of the different solutions at the same \( \tilde{L} \) and \( \tilde{q} \)

\[
\tilde{q} = \frac{\lambda^{1/4}}{\sqrt{2\pi B}} q \quad \tilde{L} = \frac{\sqrt{2\pi B}}{\lambda^{1/4}} L
\]

- Free energy is the Legendre transform of the action evaluated on solutions

\[
\mathcal{F}[\tilde{L}, \tilde{q}] \simeq S + \tilde{q} \int A_0' dr
\]
Phase diagram at zero temperature
D3/probe D5-D5 at finite temperature

- Stack of $N$ non-extremal D3-branes $\rightarrow$ Black hole in $\text{AdS}_5 \times S^5$

$$ds^2 = r^2 \left( -h(r) dt^2 + dx^2 + dy^2 + dz^2 \right)$$

$$+ \frac{dr^2}{r^2 h(r)} + d\psi^2 + \sin^2 \psi d\Omega^2_2 + \cos^2 \psi d\tilde{\Omega}^2_2$$

$$h(r) = 1 - \frac{r_h^4}{r^4}$$

- Hawking temperature $\rightarrow$ $T = \frac{r_h}{\pi} \equiv \frac{\sqrt{2} w_h}{\pi}$

$$\tilde{w}_h = \frac{\lambda^{1/4}}{\sqrt{2\pi B}} w_h$$
Finite temperature phase diagram

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Holographic double monolayer semimetal
ISSP 2016
Finite temperature phase diagram
Finite temperature phase diagram

\[ \tilde{\omega}_h = 0.5 \]

\[ \tilde{L} \]

\[ \tilde{q} \]

symmetric

inter
Summary

- **Holographic model of a double monolayer semimetal**
  - $\text{D3/probe D}5\overline{D}5$

- **Holographic mechanism for exciton condensation**
  - geometric interpretation of the intra-layer and inter-layer condensates

- **Phase diagrams at both zero and finite temperature**
  - at zero temp. $\rightarrow$ intra, inter and intra/inter
  - for small enough temp. ($w_h < 0.34$) $\rightarrow$ all four phases
  - for higher temp. $\rightarrow$ inter and symmetric
Thank you!
Backup
Graphene

- Graphene → two-dimensional material formed by carbon atoms arranged in a honeycomb lattice

- Carbon atom has four valence electrons
  - Three form strong covalent $\sigma$-bonds with neighboring atoms
  - The fourth in the $\pi$ orbital is unpaired
Graphene

- Hexagonal lattice $\rightarrow$ two triangular sub-lattices

- Band structure of graphene
Graphene

- Linearize spectrum near degeneracy points

- Relativistic dispersion relation

\[ E = \pm \hbar v_F |k| \quad v_F \simeq \frac{c}{300} \]

- Emergent Dirac equation for 4 species of massless fermion in 2+1-dim

D3 background

- Stack of $N$ D3-branes $\rightarrow$ AdS$_5 \times S^5$ background

\[ ds^2 = r^2 \left( -dt^2 + dx^2 + dy^2 + dz^2 \right) \]
\[ + \frac{1}{r^2} \left( dr^2 + r^2 d\psi^2 + r^2 \sin^2 \psi d\Omega_2^2 + r^2 \cos^2 \psi d\tilde{\Omega}_2^2 \right) \]

where $d\Omega_2^2 = d\theta^2 + \sin^2 \theta d\phi^2$ and $d\tilde{\Omega}_2^2 = d\tilde{\theta}^2 + \sin^2 \tilde{\theta} d\tilde{\phi}^2$

- It is useful to introduce other coordinates

\[ \rho = r \sin \psi \ , \quad l = r \cos \psi \]

\[ ds^2 = (\rho^2 + l^2) \left( -dt^2 + dx^2 + dy^2 + dz^2 \right) \]
\[ + \frac{1}{\rho^2 + l^2} \left( d\rho^2 + \rho^2 d\Omega_2^2 + dl^2 + l^2 d\tilde{\Omega}_2^2 \right) \]

Poincaré horizon at $r = 0 \rightarrow \rho = l = 0$
D5-$\overline{\text{D5}}$ embedding

- Embed $N_5$ D5 and $\overline{\text{D5}}$ probes in this background ($N_5 \ll N$)

- DBI + WZ actions

$$S = T_5 N_5 \left[ - \int d^6 \sigma \sqrt{- \det (g + 2\pi \alpha' F)} + 2\pi \alpha' \int C^{(4)} \wedge F \right]$$

- Worldvolume coordinates and ansatz for the embedding of the D5/$\overline{\text{D5}}$

<table>
<thead>
<tr>
<th></th>
<th>$t$</th>
<th>$x$</th>
<th>$y$</th>
<th>$z$</th>
<th>$\rho$</th>
<th>$l$</th>
<th>$\theta$</th>
<th>$\phi$</th>
<th>$\tilde{\theta}$</th>
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</tr>
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<tbody>
<tr>
<td>D3</td>
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<td>$\times$</td>
<td>$\times$</td>
<td>$\times$</td>
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</tr>
<tr>
<td>D5/$\overline{\text{D5}}$</td>
<td>$\times$</td>
<td>$\times$</td>
<td>$\times$</td>
<td>$\times$</td>
<td>$z(\rho)$</td>
<td>$\times$</td>
<td>$l(\rho)$</td>
<td>$\times$</td>
<td>$\times$</td>
<td>$-$</td>
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</tbody>
</table>

- $l$ asymptotically gives the distance between the D3 and the D5-brane $\longrightarrow$ the bare fermion mass
Worldvolume geometry of D5/\(\overline{D5}\) is for the most part determined by symmetry

- Poincaré invariance in 2+1-d \(\rightarrow\) branes wrap \(t, x, y\)
- \(\text{SO}(3)\) symmetry \(\rightarrow\) branes wrap \(S^2 (\theta, \phi)\)
- Choose \(\rho\) as the last worldvolume coordinate
- None of the remaining variables depend on \(t, x, y, \theta, \phi\)
- \(z(\rho)\) and \(l(\rho)\) are the dynamical embedding functions
- The point \(l = 0 \left(\psi = \frac{\pi}{2}\right)\) \(\rightarrow\) additional \(\text{SO}(3)\) symm.
Symmetry breaking

The geometry of the D5-D5̅ can break two symmetries

1. \( l(\rho) \neq 0 \rightarrow \text{SO}(3) \times \text{SO}(3) \rightarrow \text{SO}(3) \times \text{SO}(2) \)
   ▶ intra-layer condensate

2. \( z(\rho) \neq \text{const} \rightarrow \text{U}(N_5) \times \text{U}(N_5) \rightarrow \text{U}(N_5) \)
   ▶ partial annihilation of D5 and D5̅
   ▶ inter-layer condensate
D5-D5 embedding

- Induced metric on the D-branes worldvolume

\[ ds^2 = (\rho^2 + l^2) \left( -dt^2 + dx^2 + dy^2 \right) + \frac{\rho^2}{\rho^2 + l^2} d\Omega_2 \]

\[ + \frac{d\rho^2}{\rho^2 + l^2} \left( 1 + ((\rho^2 + l^2)z')^2 + l'^2 \right) \]

- For \( z(\rho) = \text{const} \) and \( l(\rho) = \text{const} \) → D5/D5 wv is AdS\(_4 \times S^2\)

- Charge density and external magnetic field → D5 worldvolume
gauge fields (in the \( a_\rho = 0 \) gauge)

\[ \frac{2\pi}{\sqrt{\lambda}} \mathbf{F} = a'_0(\rho) d\rho \wedge dt + b dx \wedge dy \]

\[ b = \frac{2\pi}{\sqrt{\lambda}} B \quad a_0 = \frac{2\pi}{\sqrt{\lambda}} A_0 \]
DBI action

- **DBI action for** $N_5$ D5 ($\text{D5}$)

$$S = N_5 \int d\rho \frac{\rho^2}{\rho^2 + l^2} \sqrt{(\rho^2 + l^2)^2 + b^2} \sqrt{1 + l'^2 + ((\rho^2 + l^2)z')^2 - a'^2_0}$$

where $N_5 = \frac{\sqrt{\lambda N N_5}}{2\pi^3} V_{2+1}$

- $a_0(\rho)$ and $z(\rho)$ are cyclic variables ➔ their canonical momenta are constants

$$Q = -\frac{\delta \mathcal{L}}{\delta a'_0} \equiv \frac{2\pi N_5}{\sqrt{\lambda}} q \quad q = \frac{\rho^2 a'_0 \sqrt{(\rho^2 + l^2)^2 + b^2}}{(\rho^2 + l^2) \sqrt{1 + l'^2 + ((\rho^2 + l^2)z')^2 - (a'_0)^2}}$$

$$\Pi_z = \frac{\delta \mathcal{L}}{\delta z'} \equiv N_5 f \quad f = \frac{(\rho^2 + l^2) \rho^2 z' \sqrt{(\rho^2 + l^2)^2 + b^2}}{\sqrt{1 + l'^2 + ((\rho^2 + l^2)z')^2 - (a'_0)^2}}$$

- $q = \text{charge density on the D5 (D5)}$
Equations of motion

- Solving for $a'_0(\rho)$ and $z'(\rho)$ in terms of $q$ and $f$ we get

  $$a'_0 = \frac{q(\rho^2 + l^2)\sqrt{1 + l'^2}}{\sqrt{\rho^4 (b^2 + (\rho^2 + l^2)^2)} + q^2(\rho^2 + l^2)^2 - f^2}$$

  $$z' = \frac{f\sqrt{1 + l'^2}}{(\rho^2 + l^2)\sqrt{\rho^4 (b^2 + (\rho^2 + l^2)^2)} + q^2(\rho^2 + l^2)^2 - f^2}$$

- EoM for $l(\rho)$

  $$-(l^2 + \rho^2) l'' \left(-f^2 + l^2 \left(l^2 + 2\rho^2\right) \left(\rho^4 + q^2\right) + \rho^4 \left(\rho^4 + q^2 + b^2\right)\right)$$

  $$-2 \left(l'^2 + 1\right) \left(\rho \left(f^2 + \rho^2 l^2 \left(3\rho^2 l^2 + l^4 + 3\rho^4 + b^2\right) + \rho^8\right) l' + \left(\rho^4 - f^2\right) l\right) = 0$$
Asymptotic behaviour

Asymptotic behaviour at $\rho \to \infty$ for the embedding functions $l(\rho)$, $z(\rho)$ and the gauge field $a_0(\rho)$

- $l(\rho) \underset{\rho \to \infty}{\sim} m + \frac{c}{\rho} + \ldots$
  - $m \propto$ mass term for the fermions $\implies$ we consider solution with $m = 0$
  - $c \propto$ expectation value for the intra-layer condensate

- $z(\rho) \underset{\rho \to \infty}{\sim} \pm \frac{L}{2} \mp \frac{f}{5\rho^5} + \ldots$ (for D5/$\overline{\text{D5}}$)
  - $L =$ separation between the D5 and the $\overline{\text{D5}}$
  - $f \propto$ expectation value for the inter-layer condensate

- $a_0(\rho) \underset{\rho \to \infty}{\sim} \mu - \frac{q}{\rho} + \ldots$
  - $\mu =$ chemical potential
The magnetic field $b$ can be rescaled to 1 performing the following rescalings

$$
\rho \to \sqrt{b} \rho \quad l \to \sqrt{b} l \quad z \to \frac{z}{\sqrt{b}} \quad a_0 \to \sqrt{b} a_0
$$

$$
f \to b^2 f \quad q \to bq \quad m \to \sqrt{b} m \quad c \to bc
$$

$$
L \to \frac{L}{\sqrt{b}} \quad \mu \to \sqrt{b} \mu \quad S \to b^{3/2} S
$$

$b$ disappears from all the equations. For instance, the action becomes

$$
S = \mathcal{N}_5 \int d\rho \frac{\rho^2}{\rho^2 + l^2} \sqrt{(\rho^2 + l^2)^2 + 1 \sqrt{1 + l'^2 + ((\rho^2 + l^2) z')^2}} - a_0^2
$$
Unconnected solutions

Eq. for $z(\rho)$: $z' = \frac{f \sqrt{1 + \rho'^2}}{(\rho^2 + l^2) \sqrt{\rho^4 (1 + (\rho^2 + l^2)^2) + q^2 (\rho^2 + l^2)^2 - f^2}}$

- If $f = 0$ → the solution is trivial → $z = \pm L/2$ (for D5/D5)

Unconnected solution

```
(r = 0 → l = \rho = 0)
```

“Black hole” embedding

Minkowski embedding
Connected solutions

- If \( f \neq 0 \) the solution for \( z(\rho) \) is

\[
z(\rho) = f \int_{\rho_0}^{\rho} d\tilde{\rho} \frac{\sqrt{1 + l'^2}}{(\rho^2 + l^2)\sqrt{\rho^4 (1 + (\rho^2 + l^2)^2)} + q^2(\rho^2 + l^2)^2 - f^2}
\]

- \( \rho_0 \) such that \( \rho_0^4 \left( 1 + (\rho_0^2 + l^2(\rho_0))^2 \right) + q^2(\rho_0^2 + l(\rho_0)^2) - f^2 = 0 \)

- \( z'(\rho_0) = \infty \)

- D-brane worldvolume interrupts at \( \rho = \rho_0 > 0 \)
Connected solutions

- In order to have a sensible solution we have to glue smoothly the D5/\overline{D5} solutions at $\rho = \rho_0$ $\rightarrow$ connected solution

\[ f_{D5} = f_{\overline{D5}} \text{ and } q_{D5} = -q_{\overline{D5}} \leftrightarrow \text{D5-\overline{D5} system is neutral} \]

- Inter-layer condensate exists only when the Fermi surfaces in the two layers are perfectly nested

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Holographic double monolayer semimetal

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Minkowski vs. BH embeddings

- \((f = 0, c \neq 0)\)-solutions can in principle be either BH or Mink. embeddings
- In practice if \(q \neq 0\) only BH embeddings are allowed
- Mink. embeddings \(\rightarrow\) D-brane pinches off at \(\rho = 0\) \(\rightarrow\) \(l(0) \neq 0\)
- If \(q \neq 0\) \(\rightarrow\) there must be charge sources \(\rightarrow\) F-strings suspended between the D5 and the Poincaré horizon \((r = 0)\)
- \(T_{F1} > T_{D5}\) \(\rightarrow\) strings pull the D5 to \(r = 0\) \(\rightarrow\) BH embed.
  
- For unconnected solutions \((f = 0)\) Mink. embeddings are allowed only if \(q = 0\)
## Classification of the solutions

### Scheme of the possible types of solutions

<table>
<thead>
<tr>
<th>$f = 0$</th>
<th>$f \neq 0$</th>
</tr>
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<tbody>
<tr>
<td><strong>Type 1</strong></td>
<td><strong>Type 2</strong></td>
</tr>
<tr>
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<td>connected</td>
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<tr>
<td>$l = 0$</td>
<td>$l = 0$</td>
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<td>BH embedding</td>
<td>inter</td>
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<tr>
<td>chiral symm.</td>
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<table>
<thead>
<tr>
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<tbody>
<tr>
<td><strong>Type 3</strong></td>
<td><strong>Type 4</strong></td>
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<tr>
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<td>connected</td>
</tr>
<tr>
<td>$l(\rho)$ not constant</td>
<td>$l(\rho)$ not constant</td>
</tr>
<tr>
<td>BH ($q \neq 0$)/Mink ($q = 0$)</td>
<td>intra + inter</td>
</tr>
<tr>
<td>intra</td>
<td></td>
</tr>
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</table>
D-brane separation and chemical potential

- Separation between the D5 and the $\overline{D5}$ for the connected solution $(f \neq 0)$

$$L = 2 \int_{\rho_0}^{\infty} d\rho \, z'(\rho) = \int_{\rho_0}^{\infty} d\rho \, \frac{2f \sqrt{1 + l'^2}}{(\rho^2 + l^2)\sqrt{\rho^4 (1 + (\rho^2 + l^2)^2) + q^2(\rho^2 + l^2)^2 - f^2}}$$

- Chemical potential

$$\mu = \int_{\rho_0}^{\infty} a'_0(\rho) \, d\rho = \int_{\rho_0}^{\infty} d\rho \, \frac{q(\rho^2 + l^2)\sqrt{1 + l'^2}}{\sqrt{\rho^4 (1 + (\rho^2 + l^2)^2) + q^2(\rho^2 + l^2)^2 - f^2}}$$

where, for $f \neq 0$, $\rho_0$ is the solution of

$$\rho_0^4 \left(1 + (\rho_0^2 + l^2(\rho_0))^2\right) + q^2(\rho_0^2 + l(\rho_0)^2) - f^2 = 0$$

if $f = 0 \implies \rho_0 = l(\rho_0) = 0$ for $q \neq 0$ and $\rho_0 = 0$ for $q = 0$
For the constant solution $l = 0$ the integrals can be done analytically.

- The turning point $\rho_0$ of the connected solution is

$$\rho_0 = \frac{\sqrt[4]{\sqrt{(1 + q^2)^2 + 4f^2 - 1 - \rho^2}}}{\sqrt{2}}$$

- The separation between the branes for the connected solution is

$$L = \frac{f \sqrt{\pi \Gamma \left( \frac{5}{4} \right)} \, \, _2F_1 \left( \frac{1}{2}, \frac{5}{4}; \frac{7}{4}; -\frac{f^2}{\rho_0^8} \right)}{2\rho_0^5 \Gamma \left( \frac{7}{4} \right)}$$

- The chemical potential is

$$\mu = \frac{q \sqrt{\pi \Gamma \left( \frac{5}{4} \right)} \, \, _2F_1 \left( \frac{1}{4}, \frac{1}{2}; \frac{3}{4}; -\frac{f^2}{\rho_0^8} \right)}{\rho_0 \Gamma \left( \frac{3}{4} \right)}$$
Solutions

- We must look for non-trivial (i.e. non-constant) solutions for $l(\rho)$

- EoM for $l$ is a non-linear ODE

- Numerical method to find solutions imposing the suitable asymptotic condition

  $$l(\rho) \quad \underset{\rho \rightarrow \infty}{\simeq} \quad \frac{c}{\rho} + \ldots \quad \text{massless fermions!}$$

- We used a shooting technique

- Four types of solutions are allowed
  1. $f = 0$, $c = 0$ ($z = \pm L/2$, $l = 0$) $\longrightarrow$ chiral symm.
  2. $f \neq 0$, $c = 0$ $\longrightarrow$ inter
  3. $f = 0$, $c \neq 0$ $\longrightarrow$ intra
  4. $f \neq 0$, $c \neq 0$ $\longrightarrow$ intra and inter
Example of plots of non-trivial solutions with $L \sim 1.5$ and $\mu \sim 0.77$

- $f \neq 0$, $c = 0$ → inter
- $f = 0$, $c \neq 0$ → intra
- $f \neq 0$, $c \neq 0$ → inter and intra
Solutions with zero charge density

- We are interested in solutions at fixed $L$ and $\mu$

- Eq. for $a_0$ is $a_0' = \frac{q(\rho^2 + l^2)\sqrt{1 + l'^2}}{\sqrt{\rho^4 (1 + (\rho^2 + l'^2)^2) + q^2(\rho^2 + l'^2)^2 - f^2}}$

- It has a trivial solution $a_0 = \text{const}$ when $q = 0$

- Other solutions with $q = 0$ and $a_0 = \mu$

- Among these the only relevant one $\rightarrow$

  Minkowski embedding with $f = 0$ and $c \neq 0$

  Evans, Kim [arXiv:1311.0149]
Free energy

Which configuration is favored?

- **Compare the free energies** of the different solutions at the same $L$ and $\mu$

- **The right quantity to define the free energy** is the action evaluated on solutions $\mathcal{F}_1[L, \mu] \sim S[l, z, a_0]$

$$
\delta \mathcal{F}_1 \sim \int_0^\infty d\rho \left( \delta l \frac{\partial L}{\partial l'} + \delta a \frac{\partial L}{\partial a'} + \delta z \frac{\partial L}{\partial z'} \right)' = -q \delta \mu + f \delta L
$$

$$
\mathcal{F}_1[L, \mu] = \int_{\rho_0}^\infty d\rho \frac{\rho^4 \left( 1 + (l^2 + \rho^2)^2 \right)}{\sqrt{-f^2 + q^2 (l^2 + \rho^2)^2 + \rho^4 \left( 1 + (l^2 + \rho^2)^2 \right)}} \frac{1 + l'^2}{l^2 + \rho^2}
$$

- $\mathcal{F}_1 \leftrightarrow$ **implicit function** of $L$ and $\mu$
The free energy of each solution is UV divergent. The integrand in $\mathcal{F}_1$ goes like $\rho^2$ for $\rho \to \infty$.

Regularization → subtracting to the free energy of each solution that of the trivial chirally symmetric solution (with the same $\mu$)

$$\Delta \mathcal{F}_1[L, \mu] \equiv \mathcal{F}_1[L, \mu] - \mathcal{F}_1(l = 0; f = 0)[\mu]$$

We use the regularized free energy to study the dominant configuration at fixed values of $L$ and $\mu$.

We construct the phase diagram working on a series of constant $L$ slices.
Free energy as a function of the separation: no charge

Evans, Kim [arXiv:1311.0149]
Free energy as a function of the chemical potential

\( \mathcal{F}_1 \)

- **Minkowski embedding unconnected, only intra-layer condensate**
- **Black-hole embedding unconnected, only intra-layer condensate**
- **connected \( \rho \)-independent, only inter-layer condensate**
- **connected \( \rho \)-dependent, both inter- and intra-layer condensate**

\( L = 1.5 \)

\( L = 5 \)
$(\mu, L)$–phase diagram for D3/D5-D5

- $f = 0$
- $c \neq 0$
- $q = 0$
- Mink

First order

Second order

Second order

$f \neq 0$
$c \neq 0$
$q \neq 0$
Free energy as a function of $q$ and $L$

- Compare different configurations at fixed charge density $q$ and separation $L$

- Different definition for the free energy $\rightarrow$ Legendre transform of $\mathcal{F}_1$

\[
\mathcal{F}_2[L, q] = \mathcal{F}_1 + q \mu = \\
\int_{\rho_0}^{\infty} d\rho \frac{q^2 (l^2 + \rho^2)^2 + \rho^4 \left(1 + (l^2 + \rho^2)^2\right)}{l^2 + \rho^2} \sqrt{\frac{1 + l'^2}{-f^2 + q^2 (l^2 + \rho^2)^2 + \rho^4 \left(1 + (l^2 + \rho^2)^2\right)}}
\]

- $\mathcal{F}_2$ is divergent $\rightarrow$ regularization

\[
\Delta \mathcal{F}_2[L, q] \equiv \mathcal{F}_2[L, q] - \mathcal{F}_2(l = 0; f = 0)[q]
\]
\((q, L)\)-phase diagram for D3/D5-\(\overline{\text{D5}}\)