Nonlinear Supersymmetry

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What is nonlinear supersymmetry?

- "Mismatch" of fermionic and bosonic degrees of freedom $\Rightarrow$ nonlinear realization of supersymmetry
- $\exists$ massless Goldstone fermion, the "Goldstino"
- Known cases:
  1. **Low–energy supersymmetry breaking models**
     superheavy sGoldstino $\Rightarrow$ *decouples* from the spectrum
  2. **Exact models in String Theory**
     supersymmetry: broken (bulk) vs nonlinear (brane(s))
  3. **Cosmology**
     single–field inflation
- Supervisor: Prof. I. Antoniadis, Current Projects:
  1. Constrained superfields in String Theory
  2. Partial breaking of $N = 2$ Supergravity using the constrained superfields formalism (in collaboration with Prof. I. Antoniadis and Prof. J.-P. Derendinger)

1. $N=1$ global supersymmetry
   - Kähler manifold of scalars, Kähler symmetry
   - $V = g_{\alpha\bar{\beta}} F^{\alpha} \bar{F}^{\bar{\beta}} \quad , \quad \text{SSB} \Leftrightarrow \langle 0|H|0\rangle \neq 0$
2. $N=1$ local supersymmetry
   - Kähler manifold of scalars, Kähler–**Weyl** symmetry
   - $V = g_{\alpha\bar{\beta}} F^{\alpha} \bar{F}^{\bar{\beta}} - \frac{1}{3} M \bar{M}$
   - Super–Brout–Englert–Higgs mechanism: massive gravitino

- Goldstino particle:
  - It's a fermion that transforms inhomogeneously, $\delta G^{\alpha} = \text{const} + \ldots$, constant part determined by the vevs of auxiliary fields
  - Goldstino Lagrangian ("Standard Realization") [Volkov–Akulov 1973]:

$$\mathcal{L} = -\frac{1}{2\kappa^2} - \frac{i}{2}(\lambda\sigma^{\mu}\partial_{\mu}\bar{\lambda} - \partial_{\mu}\lambda\sigma^{\mu}\bar{\lambda}) + \text{self–interactions}$$

Interactions & constraints

- Interactions:
  1. **universal** coupling to the energy–momentum tensor at low energies [Ivanov-Kapustnikov '78, Samuel-Wess '83]
  2. model–dependent couplings to SM particles [Antoniadis-Tuckmantel '04]
- Goldstino superfield: constrained chiral superfield X [Rocek–Tseytlin '78, Lindstrom–Rocek '79, Komargodski–Seiberg '09]:

$$X^2 = 0 \Rightarrow X(y, \theta) = \frac{GG}{2F}(y) + \sqrt{2}\theta G(y) + (\theta\theta)F(y)$$

- Matter couplings: $XQ = 0$, etc [Komargodski–Seiberg '09, Dall'Agata–Dudas–Farakos '16]
- Origin: $X\bar{X}Y = 0$ [Dall'Agata–Dudas–Farakos '16]
- Universality? [Dudas–von Gersdorff–Ghilencea–Lavignac–Parmentier '12, Antoniadis–Dudas–Ghilencea '12, Ghilencea '15]

Superconformal formalism

1. Extend Poincare supersymmetry → superconformal symmetry
2. Use chiral compensator superfield → $n + 1$ chiral superfields with components X^I, Ω^I, F^I
   - General Lagrangian:

$$\mathcal{L} = [\mathcal{N}(X, \bar{X})]_D + [\mathcal{W}(X)]_F + [f_{AB}(X)\bar{\lambda}^A P_L \lambda^B]_F$$

   - Kähler manifold: $G_{I\bar{J}} = \mathcal{N}_{I\bar{J}} = \frac{\partial^2 \mathcal{N}}{\partial X^I \partial \bar{X}^{\bar{J}}}$ but Kähler symmetry?
3. Break superfluous symmetries
   - D – gauge: $\mathcal{N} = -\frac{3}{\kappa^2} = -3M_P^2$
   - Projective Kähler manifold, Kähler symmetry restored

$$\mathcal{K} = -3 \ln[-\frac{1}{3}\frac{\mathcal{N}}{Y\bar{Y}}]$$

   - After the complete gauge-fixing: conventional supergravity Lagrangians

[see for example Kallosh–Kofman–Linde–Van Proeyen 2004]

The geometrical description of the coupling

- The coupling:

$$\mathcal{L} = -[(1 - X\bar{X})S_0\bar{S}_0]_D + ([(fX + W_0 + \frac{1}{2}TX^2)S_0^3]_F + \text{h.c.})$$

with Kähler potential: $K = -3\ln(1 - X\bar{X}) = 3X\bar{X}$

- Kähler–Weyl symmetry:

$$K \to K' = K - 3(X + \bar{X}), W \to W' = e^{3X}W$$

- The geometrical formulation, $\lambda = f + 3W_0$:

$$\mathcal{L}' = -[(1 + X + \bar{X})S_0\bar{S}_0]_D + ([(fX + W_0 + \frac{1}{2}TX^2)e^{3X}S_0^3]_F + \text{h.c.})$$
$$= [(-\frac{1}{2}\frac{\mathcal{R}}{S_0} + W_0 - \frac{1}{2T}(\frac{\mathcal{R}}{S_0} - \lambda)^2)S_0^3]_F + \text{h.c.}$$

[Antoniadis–CM 2015]

In terms of component fields

1. Using the constrained chiral superfield X, $X^2=0$:

$$\mathcal{L} = \int \mathrm{d}^2\Theta 2\mathcal{E}\{-\frac{3}{8}(\bar{\mathcal{D}}\bar{\mathcal{D}} - \frac{8}{6}\mathcal{R})(X\bar{X}-1) + fX + W_0\} + \text{h.c.}$$

2. Using the constrained supergravity multiplet $\mathcal{R}$, $(\mathcal{R}-\lambda)^2=0$:

$$\mathcal{L}' = -\int d^2\Theta\mathcal{E}\mathcal{R} + \int d^2\Theta 2\mathcal{E}W_0 + \text{h.c.}$$

- Gravitino mass: $m_{3/2} = |W_0|$
- Cosmological constant: 0 if $\lambda = 6W_0, \lambda = 0$ or $f = \pm 3W_0$
- Final form:

$$\mathcal{L} = \mathcal{L}' = -\frac{1}{2}eR + \frac{1}{2}e\epsilon^{abcd}(\bar{\psi}_a\bar{\sigma}_b\widetilde{\mathcal{D}}_c\psi_d - \psi_a\sigma_b\widetilde{\mathcal{D}}_c\bar{\psi}_d) \\ - eW_0\bar{\psi}_a\bar{\sigma}^{ab}\bar{\psi}_b - e\bar{W}_0\psi_a\sigma^{ab}\psi_b$$

[Antoniadis–CM 2015]

Without imposing constraints (I)

- Alternatively, consider the Lagrangian

$$\bar{\mathcal{L}} = \left[\left(-\frac{1}{2}\frac{\mathcal{R}}{S_0} + W_0 + \frac{1}{2}\rho(\frac{\mathcal{R}}{S_0} - \lambda)^2\right) S_0^3\right]_F + \text{h.c. at } \rho \to \infty$$

$$\Rightarrow \bar{\mathcal{L}} = -[b - S - \bar{S}]_D + \left([a - \frac{1}{2\rho}S^2]_F + \text{h.c.}\right)$$

- Scalar potential: $V = \frac{1}{\rho^2(b-2A_R)^2}\left\{\frac{1}{3}(A_R^2 + A_I^2)(b + A_R) - 2a\rho A_R\right\}$
- $-b \leqslant A_R < \frac{b}{2} \quad , \quad b > 0$

1. $\langle V\rangle = \langle\frac{\partial V}{\partial A_R}\rangle = \langle\frac{\partial V}{\partial A_I}\rangle = 0 \Rightarrow$ no minimum & sGoldstino does not decouple
2. $\langle\frac{\partial V}{\partial A_R}\rangle = \langle\frac{\partial V}{\partial A_I}\rangle = 0$ and $\langle V\rangle = 0$ only at $\rho \to \infty \Rightarrow$ sGoldstino does not decouple

[Antoniadis–CM 2015]

Without imposing constraints (II)

- Idea: $f(\mathcal{R})$ supergravity

$$\mathcal{L}'' = \left[\left(-\frac{1}{2}\frac{\mathcal{R}}{S_0} + W_0 + \frac{1}{2}\rho(\frac{\mathcal{R}}{S_0} - \lambda)^2 + \frac{1}{\rho}\left(S\frac{\mathcal{R}}{S_0} - F(S)\right)\right) S_0^3\right]_F + \text{h.c.} \Rightarrow$$

$$\mathcal{L}'' = -\left[(1 - \frac{1}{\rho}(S + \bar{S}))S_0\bar{S}_0\right]_D + \left\{[(W_0 + \frac{1}{2}\rho(F' - \lambda)^2 - \frac{1}{\rho}F)S_0^3]_F + \text{h.c.}\right\}$$

- Leading behaviour of the scalar potential V for $\rho \to \infty$: $V = \frac{\rho^4}{3}|F''(F' - \lambda)|^2 \to$ consider minimum for $F' = \lambda$:
  1. Scalar mass: $m_\phi = \frac{\rho^3}{3}(F'')^2$
  2. Cosmological constant: $\lambda = 6W_0$ or $\lambda = 0$
  3. SSB: $\langle|\mathcal{F}^\phi|\rangle = \left\langle\left|e^{K/2}\sqrt{g^{\phi\bar{\phi}}}\bar{D}_{\bar{\phi}}\bar{W}\right|\right\rangle \xrightarrow{\rho\to\infty} \sqrt{3}|W_0| \neq 0$,
     $m_{3/2} = \langle|W|e^{K/2}\rangle \xrightarrow{\rho\to\infty} |W_0|$

[Antoniadis–CM 2015]

1. $N = 2$ global supersymmetry
   - Hypermultiplets: hyper–Kähler manifold of scalars
   - Vector multiplets: special Kähler manifold of scalars, $\exists$ holomorphic prepotential function $\mathcal{F}(X)$
   - Using $N = 1$ superspace:
     1. 1st $N = 1$ Supersymmetry: $\delta V_i = (\epsilon Q + \bar{\epsilon}\bar{Q})V_i$
     2. 2nd $N = 1$ Supersymmetry:

$$\delta^* V_1 = (a\eta D - \bar{a}\bar{\eta}\bar{D})V_2$$

$$\delta^* V_2 = -(b\eta D - \bar{b}\bar{\eta}\bar{D})V_1$$

2. $N = 2$ local supersymmetry
   - Method: Superconformal extension with the use of two compensators, followed by suitable gauge fixing
   - Pure supergravity multiplet: graviton, two gravitini, graviphoton
   - Hypermultiplets: quaternionic–Kähler manifold of scalars
   - Vector multiplets: (projective) special Kähler manifold of scalars

Setup

- Bulk: $N = 2$ Single–Tensor multiplet (L, Φ) or (Y, χ_α, Φ) or $\mathcal{Y}$

$$\mathcal{L}_{\mathcal{Z}} = \int d^2\theta d^2\bar{\theta}\mathcal{H}(L, \Phi, \bar{\Phi})\,,$$

$$\mathcal{H}_{LL} + 2\mathcal{H}_{\Phi\bar{\Phi}} = 0$$

- Brane: $N = 2$ Maxwell multiplet (X, W) or (V_1, V_2) or $\mathcal{W}$

$$\mathcal{W} = X(y,\theta) + i\sqrt{2}\tilde{\theta}W(y,\theta) - \frac{1}{4}\tilde{\theta}^2\bar{D}^2\bar{X}(y,\theta)$$

$$\mathcal{L} = \frac{i}{4}\int d^2\theta\mathcal{F}''(X)W^2 + \frac{i}{2}\int d^4\theta\mathcal{F}'(X)\bar{X} + \text{c.c.}$$

- Coupling: Chern–Simons

$$\mathcal{L}_{CS} = -g\int d^2\theta d^2\bar{\theta}\left[LV_2 + (\Phi+\bar{\Phi})V_1\right] \text{ or } ig\int d^2\theta d^2\tilde{\theta}\mathcal{Y}\mathcal{W} + \text{c.c.}$$

Tools

- Goldstone–Maxwell multiplet:
  - deformed 2nd supersymmetry transformations

$$\delta^* X = \sqrt{2} i \eta W \, , \, \delta^* \bar{X} \sqrt{2} i \bar{\eta} \bar{W} ,$$

$$\delta^* W_\alpha = \sqrt{2} i \Big[\frac{1}{2\kappa} u \eta_\alpha + \frac{1}{4} \eta_\alpha \bar{D}^2 \bar{X} + i (\sigma^\mu \bar{\eta})_\alpha \partial_\mu X \Big]$$

$$\delta^* \bar{W}_{\dot{\alpha}} = \sqrt{2} i \Big[\frac{1}{2\kappa} \bar{u} \bar{\eta}_\alpha + \frac{1}{4} \bar{\eta}_{\dot{\alpha}} D^2 X - i (\eta \sigma^\mu)_{\dot{\alpha}} \partial_\mu \bar{X} \Big]$$

  - constraint: $\frac{1}{\kappa} X = W^2 - \frac{1}{2} X \bar{D}^2 \bar{X}$
- Add suitable Fayet–Iliopoulos terms (partial breaking [Antoniadis–Partouche–Taylor '95])

$$\mathcal{L}_{def,\mathcal{W}} = \mathcal{L}_{\mathcal{W}} - \frac{i}{4} \int d^2\theta (m \mathcal{F}'(X) + eX) + \text{c.c.} = \frac{1}{4\kappa} \int d^2\theta X + \text{c.c.}$$

$$= \frac{1}{8\kappa^2} \Big[1 - \sqrt{-\det(\eta_{\mu\nu} + 2\sqrt{2}\kappa F_{\mu\nu})} \Big] + \dots$$

$\Rightarrow$ Dirac–Born–Infeld action [Bagger–Galperin '94, Rocek–Tseytlin '01]

The result

- Setup considered and result proved in:
  Antoniadis–Derendinger–Maillard 2008,
  Ambrosetti–Antoniadis–Derendinger–Tziveloglou 2009

- Chern–Simons coupling becomes $\mathcal{YW}_{def}$

- The full theory contains the DBI action, but its coefficient is now field dependent

- Spectrum:
  1. massive vector multiplet
  2. massless chiral multiplet Φ

- How can this happen without gravity?