Axion-like particle as dark matter and source of primordial magnetic field

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Axion-like particle EFT
ALP driven magnetic field growth

**Axion-like particle: what is it?**

*Axion-like field* (ALF) — pseudo-scalar field $\theta$ with effective action

$$S[\theta] = \int d^4x \sqrt{-g} \left( L_{SM} + \frac{1}{2} (\partial \theta)^2 - V(\theta) - \frac{\theta}{f_\gamma} F_{EM} \tilde{F}_{EM} + \ldots \right),$$

where $V(\theta)$ — ALF self-interaction, $\tilde{F}_{EM} = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} F_{\alpha\beta}^{EM}$ — dual EM field tensor

For example, ALF is

- **Goldstone boson** associated with spontaneous breaking of new HEP symmetry (Peccei-Quinn, etc.);
- Effective degree of freedom in **SM plasma**, which emerges because of anomaly
Figure 1: Vacuum misalignment: a) flat potential, no mass; b) Non-zero ALP potential, the mass is generated

- Initially ALF is random Goldstone phase, $\frac{\theta_0}{\Lambda} = \text{mod}(2\pi)$ (with $\Lambda$ being new physics scale and $\theta_0$ is ALF amplitude)
- After generating the potential (at some scale $\Lambda_1 \ll \Lambda$), it becomes to oscillate in time
ALP misalignment population: particle interpretation

1) Before vacuum misalignment: \( \theta_0 \neq 0 \Rightarrow \) the ground state is the coherent state \( |\theta_0\rangle \) of massless zero modes \( \hat{a}_{1,k} \), for which the VEV of ALP operator \( \hat{\theta} \) is

\[
\langle \theta_0 | \hat{\theta} | \theta_0 \rangle = \theta_0
\]

2) After vacuum misalignment: quantization in terms of massive modes \( \hat{a}_{2,k} \). Bogoliubov transformation:

\[
\hat{a}_{2,k} = \alpha_k \hat{a}_{1,k} + \beta_k \hat{a}_{1,k}^\dagger
\]

In terms of \( \hat{a}_2 \), the VEV becomes

\[
\langle \theta_0 | \hat{\theta} | \theta_0 \rangle = \theta_0 \cos(m_\theta t - \varphi)
\]

**Conclusion**: Nonzero ALF phase \( \leftrightarrow \) set of ALPs at rest (misalignment population)
Restrictions on ALP theory parameters. Motivation

Three parameters: the mass $m_\theta$, ALP phase $\theta_0$, ALP coupling(s) $f_\gamma$. Falsifiability $\leftrightarrow$ fixing all parameters

- ALP is dark matter if
  - its lifetime $\tau_\theta$ is sufficiently large,
    $$\tau_\theta(m_\theta, f_\gamma) \geq t_{\text{Universe}}$$
  - The present coherent population energy density $\rho_\theta$ is comparable to DM energy density $\rho_{DM}$,
    $$\rho_\theta(t_{\text{present}}, m_\theta, \theta_0) = \rho_{DM}$$
- Astrophysics: conversion of photons into ALPs (with rate $\Gamma$) in stars must be small:
  $$\Gamma(\text{state parameters}, m_\theta, f_\gamma) < \Gamma_0$$

Not enough to fix all parameters!

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Pure EM theory with ALP-EM interaction:

\[ S[A, \theta] = \int d^4x \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\theta}{f_\gamma} F_{EM} \tilde{F}_{EM} \right) \]

\[ F_{EM} \tilde{F}_{EM} = \partial_\mu K^\mu, \text{ where } K_\mu = \epsilon_{\mu\nu\alpha\beta} A^{\nu} \partial^\alpha A^\beta \text{ is Chern-Simons density.} \]

Suppose external \( \theta = \theta(t) \). The term

\[ \theta \partial K \rightarrow -\dot{\theta} K_0 \]

contains only one spatial derivative, and is not positively defined \( \rightarrow \) instabilities in Maxwell equations (\textit{hep-th/0002195})!
Magnetic field growth: introduction

Simple case: $\dot{\theta} = \mu = \text{const}$. Maxwell eq. for helical magnetic field $B_{\pm} = B_x \pm iB_y$ with constant conductivity $\sigma$, for anzats $B = B(z)$:

$$\ddot{B}_{\pm} + \sigma \dot{B}_{\pm} + \left(k^2 \mp \frac{k \dot{\theta}}{f_\gamma}\right) B_{\pm} = 0$$

The growth is for helicity $B_+$, for modes

$$k \in \left(\frac{\mu}{f_\gamma} - \sqrt{\frac{\mu^2}{f_\gamma^2} - \sigma^2}; \frac{\mu}{f_\gamma} + \sqrt{\frac{\mu^2}{f_\gamma^2} - \sigma^2}\right),$$

with \textbf{extra condition} on $\mu$ (which we're looking for):

$$\frac{\mu}{f_\gamma} > \sigma$$
Assume $\theta$ as dynamical field. EOM with back reaction:

$$\ddot{\theta} = -\frac{1}{f_\gamma} \left( B_+^2 - B_-^2 \right)$$

- Stopping the growth of short wave-modes;
- Transferring the energy density to long wave-modes — inverse cascade (hep-ph/1504.04854)

Figure 2: Chiral magnetic effect. Adapted from Boyarsky 2015
Primordial magnetic field

- Is the **messenger from the Early Universe**, more earlier than the BBN
- Generates observable magnetic fields in voids or be seeds for galactic magnetic fields ([astro-ph/0207240](https://arxiv.org/abs/astro-ph/0207240))
Our task

- Dark matter
- Axion-like particle
- Primordial magnetic fields
Our analysis approach

1) Solving EOM for ALF **without** back reaction (for clarity here — with constant mass):

\[ \ddot{\theta} + 3H \dot{\theta} + m_\theta^2 \theta = 0 \Rightarrow \theta \approx \theta_0 \left( \frac{a(t_0)}{a(t)} \right)^3 \cos(m_\theta t) \]

2) Rewriting Maxwell eq. in **Mathieu-like eq.** with adiabatic coefficients \((x = m_\theta t/2, \psi_\pm = x^{1/2} B_\pm)\):

\[ \ddot{\psi}_\pm(k, x) + (A_k(x) \pm 2B_k(x) \sin(2x)) \psi_\pm(k, x) = 0 \]

3) The domain \(m_\theta, f_\gamma, \theta_0\) is determined for magnetic field growth; then (if is needed) effect of back reaction is studied
Our analysis approach

Maxwell eq. in Mathieu-like form:

\[ \psi''(k, x) + (A_k(x) \pm 2B_k(x) \sin(2x)) \psi_\pm(k, x) = 0, \]

\[ A_k(x) = \frac{1}{4x^2} + \frac{k^2}{4\alpha^2 m_\theta} \frac{1}{x}, \quad B_k(x) = \frac{\theta_0 k}{f_\gamma \alpha \sqrt{m_\theta}} \frac{1}{x^{\frac{5}{4}}} \]

Here-

\[ x = m_\theta t/2 \in (1; \infty), \quad \psi_\pm = \sqrt{x} B_\pm, \quad \alpha = 10^{-22} \text{GeV}^{\frac{1}{2}} \]

Effects of expansion of the Universe (decreasing of \( A, B \) with time): independently on \( A(1), B(1) \) system will enter stability band with time \( \to \) the growth is stopped
Preliminary results

- There is the growth (exponential) only for the negative instability zone, $2B_k(1) \gg A_k(1)$ (other zones are spoiled);
- The growth is stopped in $x_f = (A(1))^{\frac{5}{4}}$
- The growth is maximal for the mode $k_{\text{max}} \approx \frac{\alpha \sqrt{m_0 \theta_0}}{2f\gamma}$
Preliminary results

Figure 3: \(A(1) = 2, \ B(1) = 0.05\)

Figure 4: \(A(1) = 200, \ B(1) = 10\)

Figure 5: \(A(1) = 20, \ B(1) = 150\)

Figure 6: \(A(1) = 2, \ B(1) = 20\)
Conclusion

- For large growth is required that $\frac{\theta_0}{f_\gamma} >> 1$, which is **impossible** for PNG boson ALP, since

\[
\begin{align*}
\theta_0 &= \text{mod}(2\pi), \\
\Lambda &\sim \alpha f_\gamma << f_\gamma \\
\Rightarrow \theta_0 &< f_\gamma
\end{align*}
\]

- The presence of conductivity $\sigma(x)$ typically makes the condition for growth even more unrealistic:

\[
\frac{\theta_0}{f_\gamma} >> \sigma(1)
\]
Thank You for attention!