N=2* $U(1)$ 4D and 5D gauge theories in Ω background from Strings

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- $\mathcal{N} = 2^* \to \mathcal{N} = 2$, when $m \to \infty$
  $\mathcal{N} = 2^* \to \mathcal{N} = 4$, when $m \to 0$
- The system in the Ω background, characterised by two parameters ϵ_1 and ϵ_2
  $(z_1, z_2, x_5) \sim (z_1 e^{i\epsilon_1 R}, z_2 e^{i\epsilon_2 R}, x_5 + R)$
  Accompany with an $SU(2)_R$ rotation to preserve susy
  $diag(e^{iR(\epsilon_1+\epsilon_2)/2}, e^{-iR(\epsilon_1+\epsilon_2)/2}) \in SU(2)_R$.
- $\log \mathcal{Z}(\epsilon_{1,2}) = \frac{1}{\epsilon_1 \epsilon_2} \mathcal{F}$ N. A. Nekrasov, Adv. Theor. Math. Phys. **7**, no. 5, 831 (2003), [hep-th/0206161]

Motivations

- The $4D$ and $5D$ theories with massive adjoint hypermultiplet are UV finite.
- For $U(1)$ the instanton partition function has a compact form.

Gauge theory partition functions

$$\mathcal{F} = \mathcal{F}_{class} + \mathcal{F}_{1-loop} + \mathcal{F}_{inst}$$

$4D\ U(1)\ \mathcal{N} = 2^*$, $\epsilon_1 = -\epsilon_2 = \hbar$

$$\mathcal{F}^{4D}_{class} = \pi i \tau a^2 + \frac{1}{2} m^2 \log\left(\frac{m}{\Lambda}\right) - m^2 \pi i \tau - \frac{3}{4} m^2$$

$$\mathcal{F}^{4D}_{1-loop} = -\frac{\hbar^2}{12} \log\left(\frac{m}{\Lambda}\right) + \hbar^2 \sum_{g=2}^{\infty} \frac{B_{2g}}{2g(2g-2)} \left(\frac{\hbar}{m}\right)^{2g-2}$$

$$\mathcal{F}^{4D}_{inst} = (m+\hbar)(m-\hbar) \sum_{n=1}^{\infty} \frac{q^n}{n(1-q^n)}$$

$5D\ U(1)\ \mathcal{N}=2^*$

$$\mathcal{F}^{5D}_{class} = R\frac{m^3}{12} - \frac{m^2}{2}\log(R\Lambda) - m^2\pi i\tau - \frac{1}{R^2}Li_3(e^{-Rm})$$

$$\mathcal{F}^{5D}_{1-loop} = -\frac{\hbar^2}{12}\log\left(2\sinh\frac{Rm}{2}\right)$$

$$+\hbar^2\sum_{g=2}^{\infty}\frac{B_{2g}R^{2g-2}}{2g(2g-2)!}\hbar^{2g-2}Li_{3-2g}(e^{-Rm})$$

N. Nekrasov and A. Okounkov, Prog. Math. **244**, 525 (2006), [hep-th/0306238]

$$\mathcal{F}^{5D}_{inst} = \hbar^2\left(\sum_{n=1}^{\infty}\frac{q^n(1-e^{-n(m+\hbar)R})(1-e^{-n(m-\hbar)R})}{n(1-e^{-n\hbar R})(1-e^{n\hbar R})(1-e^{-nmR}q^n)}\right)$$

R. Poghossian, M. Samsonyan, J. Phys. A **42**, 304024 (2009), [arXiv:0804.3564 [hep-th]]

$\mathcal{N}=4 \to \mathcal{N}=2$ I. Antoniadis, G. D'Appollonio, E. Dudas and A. Sagnotti. Nucl. Phys. B **553** (1999) 133 [hep-th/9812118]

Give masses to the fields which are in the $\mathcal{N}=4$ vector multiplet but not in the $\mathcal{N}=2$ vector multiplet.

I. Florakis and A. Z. Assi, Fortsch. Phys. **62**, 733 (2014), [arXiv:1402.2974 [hep-th]]

In heterotic string by freely acting orbifold.

The D-brane realisation. We now sit the D-brane on a $\mathbb{C}^2/\mathbb{Z}_N$ (non-compact) orbifold singularity. As usual the $\mathbb{Z}_N$ twist is accompanied by an order N shift along the circle of radius $1/m$. An extra circle of radius R is spectator. The partition function we have for the light states

$$\mathcal{A} = \frac{1}{N}\left[\sum_{\ell=0}^{N-1} \rho\left[{0 \atop \ell}\right] \sum_{r\in\mathcal{Z}} e^{2i\pi r\ell/N} P_r(1/m)\right] \sum_{s\in\mathcal{Z}} P_s(R)\,. \qquad (0.1)$$

$$M_V^2 = \tfrac{1}{2}(Nkm)^2\,, \qquad k = 0, \pm 1, \pm 2, \ldots$$

whereas there are two hypermultiplets with masses

$$M_{H_1}^2 = \tfrac{1}{2}(1 + kN)^2 m^2\,, \qquad k = 0, 1, 2, \ldots\,,$$

and

$$M_{H_2}^2 = \tfrac{1}{2}(1 - kN)^2 m^2\,, \qquad k = 1, 2, \ldots\,.$$

Topological amplitudes

$$\mathcal{A}_g = \left\langle (V^{+}_{\rm grav})^2 \, (V^{-}_{\rm grav})^2 \, V^{2g-2}_{\rm gph} \right\rangle = g! F_g$$

I. Antoniadis, E. Gava, K. S. Narain and T. R. Taylor, Nucl. Phys. B **455**, 109 (1995), [hep-th/9507115]

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$$\mathcal{F}(\hbar) = -2 \sum_{r,s\in\mathcal{Z}} \left[\frac{1}{N} \sum_{\ell=0}^{N-1} \left(1 - \cos\left(2\pi\ell/N\right)\right) \, e^{2i\pi r\ell/N} \right] \int_0^\infty \frac{d\tau_2}{\tau_2^3} \, \frac{(\pi\hat{\hbar})^2}{\sin^2(\pi\hat{\hbar})}$$

Thank you for your attention