QFT in 1+1 with nontrivial boundary conditions

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Initial conditions

Klein-Gordon equation and boundary conditions

\[(\partial_t^2 - \partial_x^2 + m^2) \phi(t,x) = 0, \phi(t,z(t)) = 0\]

Trajectory \((t, z(t))\) of mirror is some curve in space-time. We consider timelike curves.
Formulation of the problem

- Expand the field \( \phi(t,x) \) in terms of space-time harmonics as a function of the mirror trajectory \( z(t) \)
- Quantize the field, satisfying the canonical commutation relations
- Investigate the vacuum average of tx-component of stress-energy tensor \( \langle T_{tx} \rangle \), which is the energy flux density or momentum density
- Obtain the Hamiltonian of the system, that is the operator of an evolution of the system
In this case we work in \( x \geq 0 \), so we have

**The field and boundary condition**

\[
\phi(t,x) = i \int_0^\infty \frac{dk}{2\pi} \sqrt{\frac{2}{\omega}} \sin(kx) \left[ a_k e^{-ikt} - a_k^\dagger e^{ikt} \right], \phi(t,0) = 0
\]

\[
[a_k, a_{k'}^\dagger] = 2\pi \delta(k - k')
\]

\[
[\phi(t,x), \pi(t,y)] = i \left[ \delta(x-y) - \delta(x+y) \right]
\]

where \( \pi(t,y) = \partial_t \phi(t,y) \) is canonical momentum
Stagnant mirror

Symmetrized stress-energy tensor

\[
T_{\mu\nu} = \frac{1}{2} \left( \partial_\mu \phi \partial_\nu \phi + \partial_\nu \phi \partial_\mu \phi \right) - \frac{1}{2} g_{\mu\nu} \left( \partial_\alpha \phi \partial^\alpha \phi \right), \partial^\mu T_{\mu\nu} = 0
\]

\[
H = \int_0^\infty T_{tt} \, dx
\]

\[
T_{tx} = \frac{1}{2} \left( \partial_t \phi \partial_x \phi + \partial_x \phi \partial_t \phi \right).
\]

\[
\langle T_{tx} \rangle = 0
\]

\[
H = \int_0^{+\infty} \frac{dk}{2\pi} \frac{k}{2} \left( a_k a_k^\dagger + a_k^\dagger a_k \right)
\]
Mirror with constant velocity

In this case the mirror has velocity $0 < \beta < 1$, $\phi(t, -\beta t) = 0$

The field

$$\phi(t,x) = i \int_{\gamma \beta m}^{+\infty} \frac{dk}{2\pi} \frac{1}{\sqrt{2\omega}} a_k (e^{-i\omega t - ikx} - e^{-i\omega t + ikx}) + h.c.$$  

$$[\phi(t,x), \pi(t,y)] = i \left[ \delta(x - y) - \delta(2\beta\gamma^2 t + (1 + \beta^2)\gamma^2 x + y) \right]$$

$$\omega_r = (1 + \beta^2)\gamma^2 \omega - 2\beta\gamma^2 k$$

$$k_r = 2\beta\gamma^2 \omega + (1 + \beta^2)\gamma^2 k$$
Mirror with constant velocity

\[ \langle T_{tx} \rangle = \lim_{\epsilon \to 0} \frac{1}{2} \langle \partial_t \phi(t,x) \partial_x \phi(t + i\epsilon,x) + \partial_x \phi(t,x) \partial_t \phi(t + i\epsilon,x) \rangle \]

Vacuum average of stress-energy tensor

\[ \langle T_{tx} \rangle = -\frac{1}{2\pi} \gamma^2 \beta m^2 K_0(2m\gamma(x + \beta t)), K_0\text{-McDonald function} \]

For every fixed \( x \) when \( t \to +\infty \) \( \langle T_{tx} \rangle \to 0 \)

Also boosting stagnant mirror

\[ \langle T_{tx} \rangle = \beta \gamma^2 (\langle T_{t't'} \rangle + \langle T_{x'x'} \rangle) - \langle T_{t't'} \rangle_0 - \langle T_{x'x'} \rangle_0 = -\frac{1}{2\pi} m^2 \beta \gamma^2 K_0(2mx') \]

Need to subtract vacuum terms
Mirror with constant velocity

\[ H = \int_{-\beta t}^{\infty} T_{tt} \, dx = \frac{1}{2} \int_{-\beta t}^{+\infty} [\phi_t^2(t) - \phi \phi_t^2] \, dx, \quad P = \int_{-\beta t}^{\infty} dx \, T_{tx} \]

**Evolution operator**

\[ H - \beta P = \int_{\gamma \beta m}^\infty \frac{dk}{2\pi} \frac{\gamma^2(\omega - \beta k)(\omega - \beta k - \beta (1-\beta)\omega)}{2\omega r} \left[ a_k a_k^\dagger + a_k^\dagger a_k \right] \]

Translation operator along the wall is diagonal unlike the Hamiltonian and Momentum separately

For massless field

\[ H - \beta P = (1 - \beta) \int_0^\infty \frac{dk}{2\pi} \frac{k}{2} \left[ a_k a_k^\dagger + a_k^\dagger a_k \right] \]
Conclusions

- Moving mirror violate homogeneity along the time axis, hence Hamiltonian has off-diagonal terms.
- $H - \beta P$ is diagonal operator of translations along the wall.
- Chosen method of regularization (point splitting) leads to physically sensible result.
- There are boundary delta-functions in commutation relation for field and canonical momentum.
Consider the case of "broken" mirror

Investigate $\lambda\phi^4$ interaction, i.e. corrections to Keldysh propagator
Thank you for your attention!