On the Stability of $AdS \times S$ String Vacua with broken SUSY

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International School of Subnuclear Physics - Erice, June 2017

Based on ongoing work with J. Mourad and A. Sagnotti
and on J. Mourad and A. Sagnotti, arXiv:1612.08566
1. String Vacua with Brane SUSY Breaking
2. Breitenlohner-Freedman bound
3. Perturbative Stability of these String Vacua
4. Conclusions
String Vacua with Brane SUSY Breaking

**Goal:** understand perturbative stability of String Vacua in the presence of Brane SUSY Breaking.

*(Sugimoto; Antoniadis, Dudas, Sagnotti; Angelantonj, Aldazabal and Uranga, 1999)*

Low-energy field content: $\Phi, g_{MN}, B_{p+1} + \text{Fermi}$

- We study a class of $AdS_{p+2} \times S^{8-p}$ vacua with constant dilaton
- Field strength $H = dB$ has uniform $AdS$ flux $h$ that supports the vacuum
- Linear perturbations of fields in $AdS$ have **stability bounds** for their masses.
- KK reduction: the story is unchanged in $AdS \times S$
- We provide a convenient method to compute these (Breitenlohner-Freedman) bounds

SUSY-breaking space-filling D9-branes and orientifolds generate a **'tadpole' potential** for the dilaton. Effective action:

$$S = \frac{1}{2k^2} \int d^{10}x \sqrt{-g} \left( - \mathcal{R} - \frac{1}{2} (\partial \Phi)^2 - Te^{\gamma\Phi} - \frac{e^{-2\beta\Phi}}{2(p+2)!} H_{p+2}^2 \right)$$

*(Dudas, Mourad, 2000-2001)*
Fields in $AdS_{d+1}$ have a stability bound $m^2 \geq m^2_{BF}$
(Breitenlohner, Freedman, 1982)

**Example:** free scalar field $\phi$. Metric: $ds^2 = e^{-2r/L} \eta_{\mu\nu} dx^\mu dx^\nu + dr^2$

- EOM: $\partial_r^2 \phi - \frac{d}{L} \partial_r \phi + (e^{2r/L} \Box_x - m^2) \phi = 0$
- Regular modes at $r \to +\infty$ ($AdS$ center). As $r \to -\infty$:
  $$\phi_k(r) = g_k,\omega e^{dr/2L} K_\nu(kL e^{r/L}) \sim A_k^{(\nu)} e^{(\frac{d}{2} - \nu)r/L} + B_k^{(\nu)} e^{(\frac{d}{2} + \nu)r/L}$$

The Hamiltonian, regularised on the boundary $r = L \log(\epsilon)$, diverges as $\epsilon \to 0^+$. The sign is controlled by $\nu^2 \equiv m^2 + \frac{d^2}{4}$ (in units $L = 1$)

$$H \sim \frac{\nu^2}{2} g_k^2 \int_{\epsilon}^{+\infty} \frac{ds}{s} K_\nu^2(s) \int d^{d-1}x \cos^2(k \cdot x)$$

The bound corresponds to $\nu^2 \geq 0$. This condition generalises for other fields.
Generalization: antisymmetric and symmetric (higher-spin) tensors. E.g.: \( A_M = (A_\mu, A_r) \).

- EOMs will mix \( A_r \) and \( A_\mu \). The unmixed equations have the same structure of the scalar case. For \( k = 0 \) modes
  \[
  \partial_r^2 A_\mu + \Delta_1 \partial_r A_\mu - m^2 A_\mu = 0 \\
  \partial_r^2 A_r + \Delta_2 \partial_r A_r - m^2 A_r = 0
  \]
- The boundary asymptotics \( A \sim A_+ e^{\alpha+r/L} + A_- e^{\alpha-r/L} \) suffice to compute bounds for the individual components, requiring \( \alpha_\pm \in \mathbb{R} \).
- The indices \( \nu_j^2 \equiv m^2 + \Delta_j^2/4 \) determine the bounds.
- Full EOMs for \( A_M \) are analogous to the case of coupled fields.
Consider mixed fields (possibly of various spins), arranged in a component array \( \Phi = (\varphi_j)_j \).

- Each mixing term like \((\partial^k)_{\mu_1,...,\mu_p} \varphi_i (\partial^m)_{\nu_1,...,\nu_q} \varphi_j\) has to be contracted.
- Inverse metric components \(g^{\mu\nu}\) carry a factor \(e^{2r/L}\).
- Scaling & covariance: asymptotic EOMs have the schematic form

\[
\partial_r^2 \Phi + \Delta_{(0)} \partial_r \Phi - M_{(0)}^2 \Phi \sim \sum_{q>0} e^{2qr/L} (\Delta_{(q)} \partial_r \Phi + M_{(q)}^2 \Phi)
\]

By dominant balance, the \(q > 0\) source terms that couple the equations are irrelevant for the boundary asymptotics!

- The bounds are governed by the eigenvalues of

\(N^2 \equiv M_{(0)}^2 + \Delta_{(0)}^2/4\).

Stability dictates that all the eigenvalues be non negative.
In our case, the quadratic Lagrangian for fluctuations does not generate off-diagonal $\Delta(0)$ coefficients. Asymptotically

$$\mathcal{L}^{(2)} = \mathcal{L}_{\text{kin}}^{(2)} + \frac{1}{2} M^2_{\Phi} \varphi^2 + \mathcal{O}(e^{4r/L})$$

Gauge invariance: only $\Phi = \Phi_0 + \varphi$ fluctuations develop a mass, with negligible mixing. The only condition comes from

$$M^2_{\Phi} = \gamma^2 T e^{\gamma \Phi_0} - \frac{2 \beta^2 h^2}{R^2_{AdS}} e^{2 \beta \Phi_0}$$

Every parameter is a function of $g_s$, as specified by the classical solutions. The condition one finds is $M^2_{\Phi} \geq -\frac{(p+1)^2}{4 R^2_{AdS}}$.

**Example:** $AdS_3 \times S^7$ orientifold vacuum, $p = 1$, $\beta = -1/2$, $\gamma = 3/2$

$$M^2_{\Phi} = \frac{3}{4} T g_s^{3/2} > 0$$

This vacuum is perturbatively stable.
Example: Orientifold $AdS_3 \times S^7$ vacuum

Orientifold $AdS_3 \times S^7$ vacuum, branch ($-$) | Stable

\[(m^2_\Phi - m^2_{BF})R^2_{AdS}\]

$g_s$

0 2 4 6 8 10

0 10 20 30 40 50 60 70 80 90
We are developing a systematic method to study perturbative stability in $AdS \times S$ field theories, in particular for those arising from orientifolds with Brane SUSY Breaking.

- In the cases we analysed mixings of fluctuations do not affect stability
- Computations reduce to matrix diagonalization
- The ‘$-$ branch’ of the $AdS_3 \times S^7$ orientifold solution, which has a perturbative corner (large $R_{AdS}$, small $g_s$), is also perturbatively stable in its whole range of parameters
Future work:

- An interesting next step would be to add a non-abelian gauge field $A$
- Theories with non-negligible derivative mixings may change BF bounds in non-trivial ways
- Non-perturbative stability analysis (e.g. instantons)