BPS Black holes in AdS and a magnetically induced quantum critical point

A. Gnecehi
Outline

Motivations

Supersymmetric Black Holes

Thermodynamics and Phase Transition

Conclusions and Outlook
Black holes as a Quantum Gravity lab

- String Theory is a framework that provides a UV completion of a unified theory of GR and QFT
- AdS/CFT is a tool to investigate quantum gravity

Two main areas of investigations

1. Supersymmetric black holes, microscopic derivation
2. Black holes in Anti de Sitter spacetime

Fig. from McGreevy, 2009
Holographic applications

- Black hole physics can teach about strongly coupled field theories
  - Investigate quantum critical phases of strongly coupled solid state systems

Hawking-Page transition for a black hole in anti de Sitter spacetime [‘83]

⇒ Holographic interpretation as confinement/deconfinement phase transition
  Witten [‘98]

- Goal: construct an analytical example from black hole thermodynamics
  - In the dual field theory, states can undergo phase transitions
  - Study the phase space of the black hole solutions
Einstein-Maxwell theory

\[ e^{-1} \mathcal{L} = -\frac{1}{4} R + \frac{1}{4} F_{mn} F^{mn} + \frac{1}{2} \Lambda \]

Equations of motions

\[ G_{mn} = T_{mn} + g_{mn} \Lambda, \quad \partial_m (eF^{mn}) = 0. \]

The black hole solution is

\[ ds^2 = -V dt^2 + \frac{dr^2}{V} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \]

\[ F = \frac{Q}{r^2} dt \wedge dr + H \sin \theta d\theta \wedge d\phi \]

with warp factor

\[ V(r) = 1 - \frac{2M}{r} + \frac{Z^2}{r^2} - \frac{\Lambda}{3} r^2 \]
Black holes and Supersymmetry

The solution is a supersymmetric solution of N=2 minimal gauged Supergravity

\[ e^{-1} \mathcal{L} = -\frac{1}{4} R + \frac{1}{2} \bar{\psi}_m \gamma^{mnp} \mathcal{D}_n \psi_p + \frac{1}{4} F_{mn} F^{mn} + \]
\[ + \frac{i}{8} \mathcal{F}^{mn} \bar{\psi}_p \gamma_{[m} \gamma^{pq} \gamma_{n]} \psi_q - \frac{1}{2} g \bar{\psi}_m \gamma^{mn} \psi_n - \frac{3}{2} g^2 \]

Gravitini are minimally coupled

\[ \mathcal{D}_m = \nabla_m - igA_m \]

SUSY equations

\[ \delta e^a_m = \text{Re}(\bar{\epsilon} \gamma^a \psi_m) \], \[ \delta \psi_m = \hat{\nabla}_m \epsilon \], \[ \delta A_m = \text{Im}(\bar{\epsilon} \psi_m) \]

- Electric solutions preserve \( \frac{1}{2} \) of the original supersymmetry
- Magnetic solutions preserve \( \frac{1}{4} \) of the original supersymmetry, but all have naked singularities at \( r = 0 \)
BPS black holes: scalar fields to the rescue

- Bosonic sector of $\mathcal{N} = 2$ Supergravity Lagrangian

$$S = \int d^4 x \left( -\frac{R}{2} + g_{ij} \partial^i z^i \partial^j \bar{z}^j + \text{Im} \mathcal{N}_{\Lambda \Sigma} F^\Lambda_{\mu \nu} F^{\Sigma \mu \nu} + \frac{1}{2\sqrt{-g}} \text{Re} \mathcal{N}_{\Lambda \Sigma} \epsilon^{\mu \nu \rho \sigma} F^\Lambda_{\mu \nu} F^{\Sigma}_{\rho \sigma} - V_g \right)$$

- BPS attractors:
  - Ungauged Supergravity
    $$\text{AdS}_2 \times S_2 \leftrightarrow \mathbb{R}^{1,3}$$
    $$\partial_i | \mathcal{Z}(q, p, z^i, \bar{z}^i) |_{\text{hor}} = 0$$
    [Ferrara, Gibbons, Kallosh, Strominger, ‘95-’96]
  - Gauged Supergravity
    $$\mathcal{Z} \equiv \langle Q, \mathcal{V} \rangle = L^\Lambda q^\Lambda - M^\Lambda p^\Lambda$$
    $$V_{BH} = |D \mathcal{Z}|^2 + |\mathcal{Z}|^2$$
    $$\mathcal{L} = \langle \mathcal{G}, \mathcal{V} \rangle = L^\Lambda g^{\Lambda} - M^\Lambda \tilde{g}^\Lambda$$
    $$V_g = g^{i\bar{j}} D_i \mathcal{L} D_{\bar{j}} \mathcal{L} - 3|\mathcal{L}|^2$$
    $$\partial_i \left| \frac{\mathcal{Z}}{\mathcal{L}} (q, p, z^i, \bar{z}^i) \right|_{\text{hor}} = 0$$
    [Dall’Agata, A.G., ‘11]
Thermodynamic ensemble

Euclidean path integral formulation of gravity at the semiclassical:
[Gibbons, Hawking ‘76, York, ‘86]

\[ Z = \int d[g_{\mu\nu}] d[\phi] \exp\{iI_e[g_{\mu\nu}, \phi]\}. \]

For a system like black holes and black branes, exhibiting a thermodynamic behaviour, the partition function defines a free energy, which, within a saddle point approximation, corresponds to the Euclidean on-shell action

\[ -\beta F = \ln Z = iI_e[g^*, \phi^*], \]

with \( \beta = T^{-1} \).

⇒ What are the thermodynamic variables?
Thermodynamic ensemble

First law

\[ dM = TdS + \chi^\Lambda dq^\Lambda + \phi^\Lambda dp^\Lambda \]

- **Electric configuration.** For a system with \( n \) charges \( q^\Lambda (\Lambda = 1, \ldots, n) \)

  \[ F(T, \chi) = M - TS - q^\Lambda \chi^\Lambda \]

- **Magnetic configuration.** For a system with \( n \) charges \( p^\Lambda (\Lambda = 1, \ldots, n) \)

  \[ F(T, p^\Lambda) = M - TS \]

Adding boundary terms on the action change the boundary conditions: **Léandre transformations**, change the ensemble. [Hawking, Ross, ’95]

In Supergravity, that corresponds to an **electric-magnetic duality rotation**
Black brane in $\text{AdS}_4$

[Klemm, Vaughan, Hristov, Toldo, Vandoren, ’10 - ‘12]

Specify a theory of $\mathcal{N} = 2$ Supergravity with gauging:

$$\mathcal{L} = \int \sqrt{-g} \, d^4x \left( -\frac{R}{2} + \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi + I_{\Lambda\Sigma} F^\Lambda_{\mu\nu} F^\Sigma_{\mu\nu} - V_g(\phi) \right)$$

with fixed

$$V_g(\phi) = -\frac{3}{\ell_{\text{AdS}}^2} \cosh \left( \sqrt{\frac{2}{3}} \varphi(r) \right), \quad \ell_{\text{AdS}}^2 = \frac{1}{g^2}.$$

The black brane solutions have metric

$$ds^2 = -\frac{f(r)}{\sqrt{H_0(r)H_1^3(r)}} \, dt^2 + \sqrt{H_0(r)H_1^3(r)} \left( \frac{dr^2}{f(r)} + r^2(dx^2 + dy^2) \right)$$

where

$$H_0(r) = 1 - \frac{3b}{r}, \quad H_1(r) = 1 + \frac{b}{r}, \quad f(r) = \frac{c_1}{r} + \frac{c_2}{r^2} + r^2 H_0(r) H_1(r)^3,$$

the other bosonic fields are

$$F^0 = \frac{q}{2(r - 3b)^2} dr \wedge dt, \quad F^1 = \frac{B}{2} dx \wedge dy, \quad e^{\sqrt{8/3}\varphi} = \frac{r + b}{r - 3b}.$$
Good singularity

There exists a competing solution in phase space

- Black brane limit in which the horizon coincides with the singularity: "good" singularity [Gubser, 2000].

\[ g_{tt}(r_h) = 0, \quad \text{as} \quad r_h \to r_s \]

- \( g_{xx} = g_{yy} = \sqrt{(r - 3b)(r + b)^3} \)

- Family of black branes with horizon \( r_h = 3b + \epsilon \)

\[ g_{tt}(3b + \epsilon) = 0 \]

- For epsilon \( \epsilon \ll 1 \)

\[ |B| = 8\sqrt{2}b^2 + \frac{(6b^2 + \chi^2)\epsilon}{\sqrt{2}b} + \mathcal{O}(\epsilon^2) \]
Phase transition

What solution dominates?

\[ \Delta F = F_{BB} - F_{TG} = \frac{27B^2 + 32\chi^4}{24\sqrt{6}|\chi|} - 2^{-\frac{1}{4}}|B|^{\frac{3}{2}}. \]

There is a second order phase transition at

\[ |B_c| = \frac{4\sqrt{2}}{3}\chi^2, \]

between a gapless (black brane) and a gapped phase (thermal gas).

No confinement-deconfinement phase transition, however.
Conclusions and Outlook

- Static supersymmetric black holes in $AdS_4$ have been constructed and analyzed
- Black hole attractors for gauged Supergravity have an interesting interpretation in the dual field theory
- Thermodynamics and electric magnetic duality are interconnected and reveal a nontrivial phase space
- A quantum critical point emerges from the study of good singularities

To do:
- Attractors for rotating black holes in Anti de Sitter
- Study phase space at finite temperature
- Supersymmetry and holographic renormalization
- Uplift to String/M-theory for generic gaugings to resolve good singularity
Thank you!
BPS black holes in $AdS_4$

- Solutions of $\mathcal{N} = 2$ $U(1)$-gauged Supergravity, a truncation of $SO(8)$-gauged $\mathcal{N} = 8$ supergravity [de Wit, Nikolai, ‘82, Duff & Liu, ‘99].

Interpolating geometry and scalar fields with nontrivial radial profile

$$ds^2 = -e^{2U(r)}dt^2 + e^{-2U(r)}(dr^2 + e^{2\psi(r)}d\Omega^2)$$

[Cacciatori, Klemm, ‘09]

- Two main features wrt asymptotically flat solutions
  - Magnetically charged $p^\Lambda g_\Lambda = \kappa$
  - Dirac quantization condition between gravitino and black hole charges
  - Preserve a smaller amount of Supersymmetry: $\frac{1}{4}$-BPS solutions
    [A.G., Dall’Agata - Hristov, Vandoren ‘11]

- They can be obtained as a compactification of M-theory on $S^7$, and asymptote to $AdS_4 \times S^7$. Charges and fluxes are then interpreted as M-branes constituents, they corresponds to BPS states in the dual ABJM theory.

[Aharony at al. ‘08]
BPS black holes in $AdS_4$: attractors

ungauged Supergravity

$AdS_2 \times S_2 \leftrightarrow \mathbb{R}^{1,3}$

$\partial_i |Z(q, p, z^i, \bar{z}^i)|_{\text{hor}} = 0$

to gauged Supergravity

$AdS_2 \times S_2 \leftrightarrow AdS_4$

$\partial_i \mathcal{W}|_{\text{hor}} = 0 \quad \partial_i \mathcal{L}|_{\text{hor}} = 0$

Translates radial equations in algebraic relations that capture the physics of the (dual) theory

A new quantity plays the role of superpotential

$\mathcal{W} = e^U |Z - i e^{2(\psi - U)} \mathcal{L}|$

New attractor equations

$\partial_i \mathcal{W}|_h = 0, \quad \mathcal{W}|_h = 0 \quad \rightarrow \quad e^{2A} = -i \frac{Z}{\mathcal{L}} = R_5^2$

[A.G&G. Dall’Agata, ‘11]

Holographically, this quantity corresponds to a field theory partition function in the dual ABJM theory to $N=8$ Supergravity.

$\Rightarrow$ Microscopic counting of black hole states in $AdS_4$ by Zaffaroni, Hristov, Benini by compactifying ABJM on $S^2 \times S^1$
Phase transition

Black brane

\[ F_{bb} = M_{bb} - TS_{bb} + q_{bb} \chi_{bb}, \]

- \( M \) is the mass of the black brane,
  \[ M = \frac{B^2 - q^2}{4b}, \]
- Thermodynamic potential
  \[ dF_{bb} = -S_{bb}dT + q_{bb}d\chi_{bb} + m_{bb}dB \]

The magnetization is qualitatively different for BB and TG

\[ m_{bb} = \left. \frac{\partial F_{bb}}{\partial B} \right|_{\chi, T} = 3\sqrt{\frac{3}{2}} \frac{B}{|\chi|}, \]

\[ m_{TG} = 3 2^{-\frac{5}{4}} \sqrt{|B|} \text{sgn}(B). \]

Thermal gas

\[ b_{TG} = +2^{-\frac{7}{4}} \sqrt{|B|}. \]

- The free energy for the thermal gas is a function of \( B \) only, at any temperature
  \[ F_{TG} = M_{TG} = \frac{B^2}{4b_{TG}} = 2^{-\frac{1}{4}} |B|^{3/2}, \]
  \[ dF_{TG} = m_{TG}dB \]