Particles on the rotating channels in the wormhole metrics

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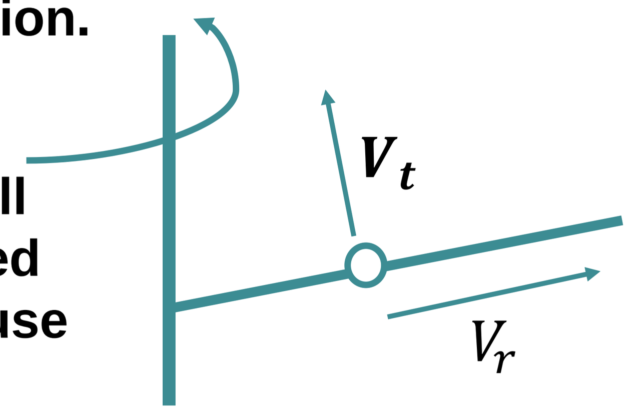
Introduction

Before we consider actual problem emphasizing one fact would be useful. Imagine we have got a stick which rotates in the horizontal plane with constant angular velocity. A bead can slide on the stick without friction.

As soon as the bead gets far enough its tangential velocity will get closer to the speed of light which will cause its radial velocity to start deceleration.



When the tangential speed will become equal to the speed of light its radial velocity will become zero. This interesting fact might have an important role in many astrophysical phenomena. Based on this fact we will study the dynamics of particle on the rotating channels in the wormhole metrics.

In the scientific literature the possibility of the existence of wormholes (WH) is actively discussed. It is worth noting that some authors consider that certain massive astrophysical objects may be entrances to WHs, which in turn might be loaded by extremely strong magnetic fields of the order of $10^{13}G$ or even higher.(Ref. 5).In such an enormous value of magnetic field gyroradius of ultrarelativistic protons ($\gamma \sim 10^7$) :

gyroradius≈1cm.

Particle will follow the magnetic field lines.

Our aim:

Aim of the paper, which is basis of this poster, was to study the dynamics of particles sliding along rotating magnetic field lines in the WH metrics. We examine the most simplified Ellis WH metrics and considered the dynamics of particles to understand conditionss under which particles might penetrate the inner region of the WH and freely leave it.

Main consideration

As it has already been mentioned, we examine the simplified Ellis Wormhole metrics which can be written as:

$$ds^2 = -dt^2 + dl^2 + (b_0^2 + l^2)(d\theta^2 + \sin^2\theta)\, d\phi^2)$$

- $time - t$
- $\theta, \phi \ - spherical\ coordinates$
- $l - coordinate\ measuring\ proper\ radial\ distance$
- $b_0 \ - wormole\ constant$

Because the particle is on a magnetic field line and the rotation of this field line causes a change in the azimuthal coordinate, we get:

$$\Phi = \phi(l) + \omega t$$

Taking this into account we will get following expression for the wormhole metrics:

$$g_{\alpha\beta} = \begin{pmatrix} -1 + \omega^2 \sin^2\theta(l)(b_0^2 + l^2) & \phi'(l)\omega\sin^2\theta(l)(b_0^2 + l^2) \\ \phi'(l)\omega\sin^2\theta(l)(b_0^2 + l^2) & 1 + (\theta'(l)^2 + \sin^2\theta(l)\phi'(l))[b_0^2 + l^2]) \end{pmatrix}$$

Using the Euler-Lagrange equation after constructing the Lagrangian, from metrics above, we will get the conserved quantity of energy:

$$E = -\gamma(g_{00} + g_{01}v) = const$$

Where $\gamma = (-g_{00} - 2g_{01}v - g_{11}v^2)^{-\frac{1}{2}}$

After straightforward calculations Lagrange's equations lead to an expression for the radial acceleration. This time, to make it simple we only consider two main cases:

- $a = 0; \theta = const$

$$\frac{dl^2}{dt^2} = \frac{\omega^2 l \sin^2\theta}{1 - (b_0^2 + l^2)\omega^2 \sin^2\theta}[1 - (b_0^2 + l^2)\omega^2 \sin^2\theta \ - 2v^2]$$

Acceleration will be negative from very beginning if:

$$v > \left(\frac{1 - b_0^2\omega^2\sin^2\theta}{2}\right)^{1/2}$$

And $l_{LC} = \left(\frac{1}{\omega^2 \sin^2\theta} - b_0^2\right)^{\frac{1}{2}}$

Now lets consider a more complicated and interesting case:

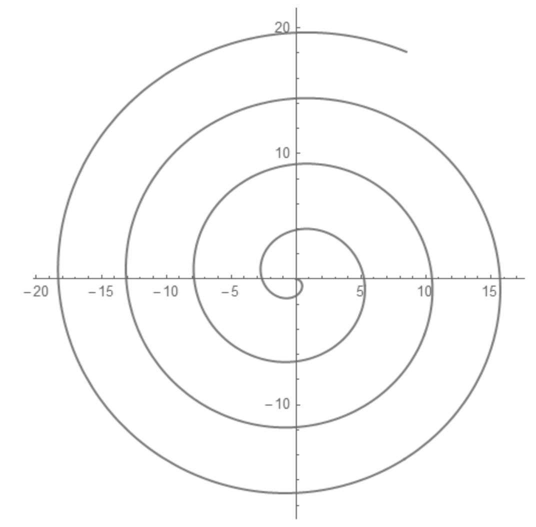
- $\boldsymbol{\phi = al; \theta = const}$ **(Archimedean Spiral)**

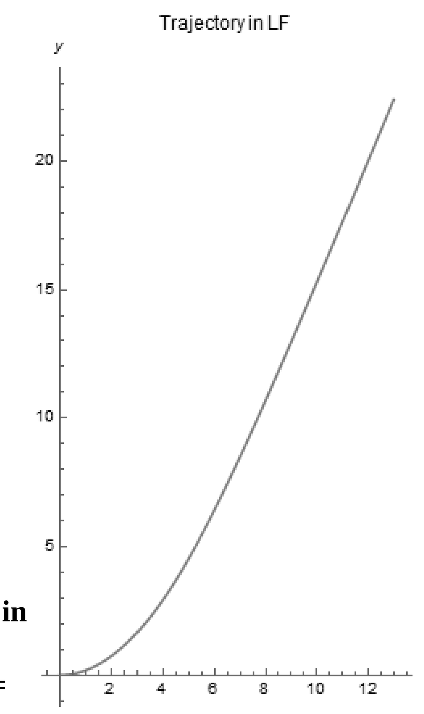
$$\frac{dl^2}{dt^2} = -\frac{l\Omega sin^2\theta[va-\omega+2v^2\omega+(b_0^2+l^2)\omega\Omega^2 sin^2\theta]}{1+(b_0^2+l^2)(a^2-\omega^2)sin^2\theta}$$

$$\boldsymbol{\Omega = \omega + av}$$

To make it easy to imagine the dynamics of the particle we will introduce numerical solutions for the trajectory as well as for the velocity in a local and a lab frame of reference.

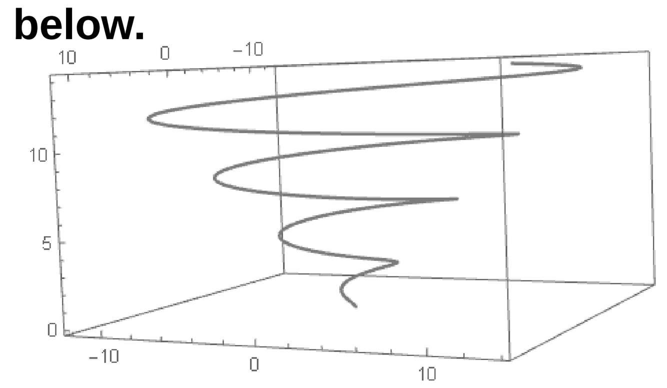
Discussion

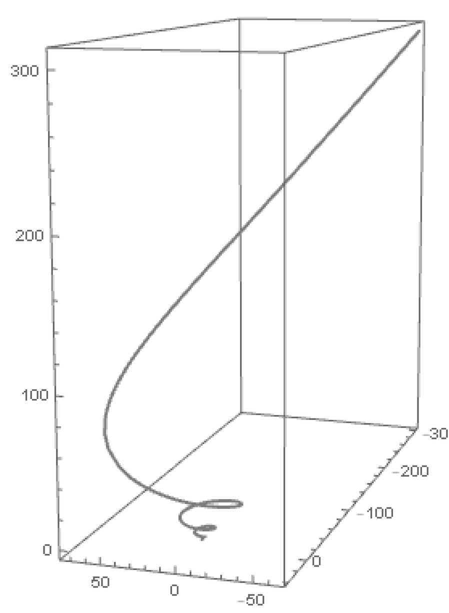




These two figures show us the trajectory in 2D, in a local frame of reference and in a laboratory frame of reference. $\omega = 1; \theta = \frac{\pi}{2}; b_0 = 0.1 ; l_0 = 0; v_0 = 0.99; a = -1.2$

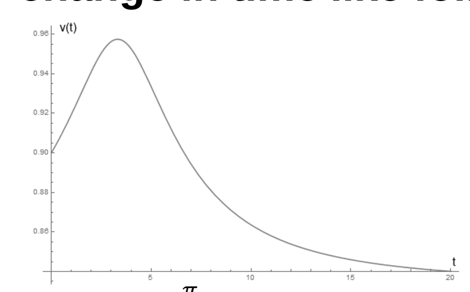
Because we took the conditions when the particle can leave the wormhole, it will reach the force free regime and leave the wormhole while moving on a straight line. The particle moves on the straight line in 3D as well. These are shown below.

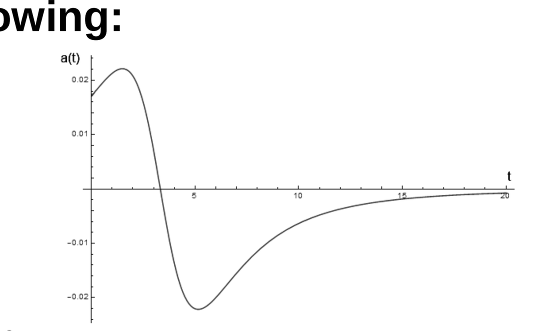




Those graphs just show the trajectory of the particle but in the 3D. $\omega = 1; \theta = \frac{\pi}{4}; b_0 = 0.1 ; l_0 = 0; v_0 = 0.85; a = -1.2$

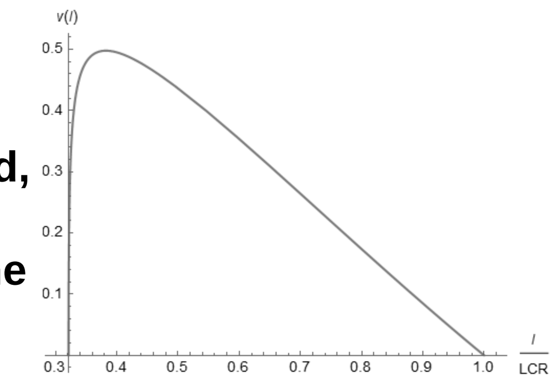
During This kind of motion velocity and acceleration will change in time like following:





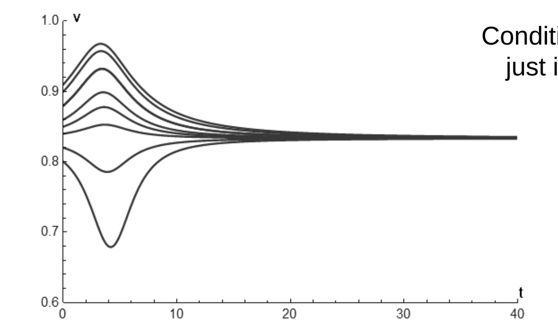
$\omega = 1; \theta = \frac{\pi}{2}; b_0 = 0.1 ; l_0 = 0; v_0 = 0.99; a = -1.2$

If the conditions for the particle to leave the wormhole are not setisfied, the following will happen: the particle will stop on the LCR. In this situation the particle's velocity profile depends on the initial velocity but the general face of the velocity is shown on the graph on the right.



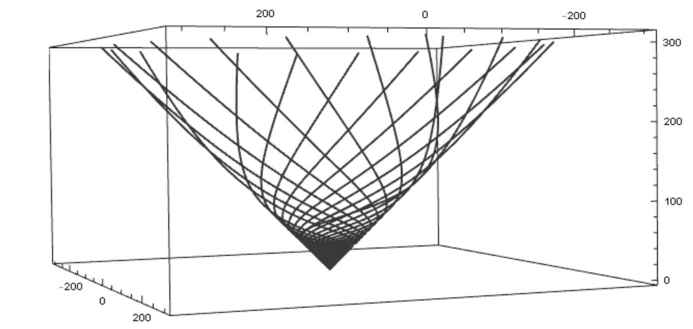
$\omega = 1; \theta = \frac{\pi}{2}; b_0 = 0.1 ; l_0 = 0.3R_{lc}; v_0 = 0; a = -1.2$

It is worth noting that main condition, for particle to leave the wormhole with the constant speed, is $-\frac{\omega}{a} < 1$. If this condition is satisfied and initial velocity is less than critical, meaning $v_0 < -\frac{\omega}{a}$, then we will get all the final velocities same $v = -\frac{\omega}{a}$. This is shown on the next graph.



Conditions are all the same just initial velocities are different:

Finally if where were a few particles satisfying the conditions to leave the wormhole we would see the following image:



Conclusions

- **We have developed a method for studying particle dynamics in rotating 2D and 3D trajectories imbedded in the simplest Ellis-Bronnikov WH metrics to understand conditions when the particles might potentially leave the interiors of WHs.**
- **For this purpose we have rewritten the metrics on a prescribed trajectories (field lines) and derived the equations of motion, governing dynamics of particles.**
- **As a first example we have studied motion on straight rotating field lines. It has been shown that depending on initial conditions the particles either radially accelerate or decelerate from the very beginning of motion. The process has been analysed also by means of the effective potential and it was found that in this case particles will never leave the WH.**
- **We have found that if dynamics is studied on the 2D or 3D Archimedes' spirals, under certain conditions the particles might asymptotically reach infinity, becoming force free. On the other hand, it has been shown that for a certain class of Archimedes' spirals, like the straight field lines, the particles will remain inside the WH interior.**

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*LCR-Light cylinder radius