Unconventional Supersymmetry: From Supergavity to Graphene

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Abstract

In the spirit of the holographic correspondence, we have investigated the relation between a D = 3 + 1 AdS supergravity (SGRA) theory in the presence of a boundary of space-time deformed in J. Anchordoqui, D. A. Aura, Nucl. Phys. B 825 (2010) 426-443, arXiv:0909.3057 [cond-mat. strong-interact.] and a model (2D) presented in J. Ac- quard, M. Visser, J. Zinn-Justin, Nucl. Phys. B 125 (2002) 079, arXiv:0503596 [hep-th] for a charged scalar 1-form in a 3 + 1 dimensions with OSp(2|2) symmetry selecting a Super-Chern-Simons (SCS) Lagrangian, which can describe some condensed matter systems with fermionic excitations in 2 + 1 dimensions, like graphene. We have found that the constraints on the 3D boundary of D = 3 + 2 SGRA can be recovered as equations of motion from a 3D SGRA with OSp(2|2) (SO(2,1)) invariance, whose OSp(2|2) invariant part coincides with the SCS Lagrangian, while the remaining part decouples. Indeed, a model where this can be explicitly realized by means of an appropriate choice of the boundary conditions is provided by the "asymptotically limiting" theory. As an example of a model where this can be explicitly realized, a model with an AdS/Keer black hole, an asymptotically-AdS solution with an AdS3 geometry at the boundary.

The graphene Dirac cone

The Electron Band Structure of graphene

At the Dirac points (for a range of 10 eV) the spectrum is linear:

\[ \epsilon = v_F k \]

Due to the interaction of the electrons with the lattice atoms, usually \( \epsilon < v_F \).

Graphene

The spinor 1–form associated with the spinorial 1–form is a \( \psi_A \) \( \gamma^A \bar{\psi}_R \) (\( A = 0,1,2,3 \)).

\[ \psi_A = \psi_1 + i \psi_2 + i \psi_3 + i \psi_4 \]

The powerful methods of the relativistic theories turn out to be a very useful tool for exploring the special properties of graphene possessing a two-dimensional spatially curved surface. In the presence of a space-time defect, such as a boundary of the AdS3, or wormholes in bilayer graphene (sites A and sites B).

Manifolds (22) and (23) covariant differential.

\[ A_{\mu} \rightarrow A_{\mu} + \ell \omega_x \]

Denote by:

\[ \psi = \psi_1 + i \psi_2 + i \psi_3 + i \psi_4 \]

Explicit D = 3 description: Asymptotic "ultraspinning" limit. Want: local AdS as effective theory on boundary \( \partial \text{Mat} \) \( \rightarrow \infty \).

4. Comparison with "unconventional" supersymmetry

3. Explicit D = 3 Description: Asymptotic "ultraspinning" limit

2. Boundary behavior of N=2, AdS 4 supergravity

1. Graphene and the Dirac equation

\[ \chi \equiv \psi_1 + i \psi_2 - i \psi_3 - i \psi_4 \]

Boundary behavior of N = 2, AdS 4 supergravity

Asymptotic behavior of the N=2, AdS 4 supergravity

Comparison with "unconventional" supersymmetry

Plan

15. Summary: Our results

14. Outlook

13. Conclusion

12. The AdS/CFT correspondence

11. Summary of results

10. The AdV model displays N = 2 local supersymmetry

9. With J. Zanelli we are working on the addition of the spinorial 1-form to the Dirac equation.

8. For a charged scalar 1-form in a 3 + 1 dimensions with OSp(2|2) symmetry selecting a Super-Chern-Simons (SCS) Lagrangian, which can describe some condensed matter systems with fermionic excitations in 2 + 1 dimensions, like graphene.

7. The spinor 1–form associated with the spinorial 1–form is a \( \psi_A \) \( \gamma^A \bar{\psi}_R \) (\( A = 0,1,2,3 \)).

6. In the presence of a space-time defect, such as a boundary of the AdS3, or wormholes in bilayer graphene (sites A and sites B).

5. In the spirit of the gauge/gravity correspondence, which relates a gauge theory in D dimensions to a gravity theory in dimension one higher, we want to relate a Super-Chem-Simons Lagrangian yielding the graphene Dirac equation in D = 2 + 1 to an AdS3 supergravity.

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3. Explicit D = 3 Description: Asymptotic "ultraspinning" limit

2. Boundary behavior of N=2, AdS 4 supergravity

1. Graphene and the Dirac equation

\[ \psi = \psi_1 + i \psi_2 + i \psi_3 + i \psi_4 \]

Explicit D = 3 description: Asymptotic "ultraspinning" limit. Want: local AdS as effective theory on boundary \( \partial \text{Mat} \) \( \rightarrow \infty \).

Fefferman-Graham parametrization: coordinates \( x^A \), \( \mu = 0,1 \), at split into \( x^0 \), \( \mu = 0,1 \) on \( \partial \text{Mat} \), radial coordinate \( r = r \).

Lorentzian SO(1,3) \times SO(1,2) 1-forms:

\[ \chi^{(1,1)} \equiv \chi^{(0,1)} \chi^{(1,0)} \]

Gravitino: \( \Psi_{\alpha}^{(A)} = \Psi_{\alpha}^{(B)} + \Psi_{\alpha}^{(C)} \)

"ultraspinning" limit of AdS3/Keer black-hole M. Cottol.

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Plan

1. Graphene and the Dirac equation

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5. Conclusions and outlook

Some remarks

-- In the AdV model the supersymmetry algebra is realized as a gauge symmetry with fields in the adjoint representation.

-- The number of fermions does not coincide with the number of bosons: it is an "unconventional" supersymmetry.

-- The spacetime dreibein \( e^\mu \) of the AdV model does not coincide with the super-pants dreibein \( E^\mu \). Consistency requires:

\[ E^\mu = M(\chi)^{-1} e^\mu \]

\[ \chi \equiv \psi_1 + i \psi_2 - i \psi_3 - i \psi_4 \]

With J. Zanelli we are working on the addition of the spinorial 1-form to the Dirac equation.

-- The spinor \( \chi \) is the spin projection of the \( D = 2 + 1 \) gravitino \( \psi_1 + i \psi_2 - i \psi_3 - i \psi_4 \) on the other hand:

\[ \chi_\ell = \chi^{(1)} + \chi^{(1)} \]

The spinor \( \chi_\ell \) is originating from the radial component of the D = 3 + 1 gravitino field.

-- The Klein paradox, i.e. transmission of an electron across a barrier in graphene. (J. Anecd.)

-- Transmission coefficient

Graphene: transparent

Biayer graphene: on the supergravity side, could it correspond to wormholes between two disjoint 2 + 1 boundaries of 3 + 1 space-time?

Ordinary semiconductors: exponential du-


-- Effect of doping on graphene: impurities act as conical singularities and generate non-trivial holonomies.