CONDENSATE OF μ-BOSE GAS AS A MODEL OF DARK MATTER

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ABSTRACT

Very popular Cold Dark Matter (CDM) model is in a good agrement with observational data on the large scales, while on the small (< 1Mpc) scales it faces some difficulties. As it was shown in [1], Bose-Einstein condensate dark matter (BEC DM) model solves core-cusp problem, which arises within CDM. In our work, we extend BEC DM model by replacing it with μ-deformed Bose gas, whose particles obey non-standard statistics. Within μ-Bose gas model, dependence of thermodynamical functions on the deformation parameter μ arises through so-called μ-calculus. Using thermodynamical geometry, we prove existence of phase transition of Bose-condensation type in the studied system, and show that the condensation temperature T_c^μ is higher than standard Bose T_c. We also find some parameters of DM halo, and show that their dependence on the parameter μ enables to treat weak points of BEC DM model.

The presented results are published in[3].

μ – CALCULUS

To "μ-deform" Bose gas thermodynamics, we use so-called μ-calculus. We extend usual derivative by replacing it with "μ-derivative" which is defined (see Ref. [2]) by its action on monomials:

$$\mathcal{D}_x^{(\mu)} x^n = [n]_\mu x^{n-1} ,$$

where $[n]_\mu$ denotes the μ-bracket that is

$$[n]_\mu \equiv \frac{n}{1+\mu n}.$$

Below we will need the μ-polylogarithm – it generalizes usual polylogarithm so that

$$g_l^{(\mu)}(z) = \sum_{n=1}^{\infty} \frac{[n]_\mu}{n^{l+1}} z^n. \quad (1)$$

DM HALO PARAMETERS

We suggest DM-halo radius to be similar to that in familiar BEC DM model,

$$R = \pi\sqrt{\frac{\hbar^2 a}{Gm^3}}.$$

Then we can infer total mass of dark matter halo

$$M^{(\mu)} = \frac{\pi}{6} m g_{3/2}^{(\mu)}(1) f^3,$$

where by f we denoted the factor

$$f = \sqrt{\frac{2\pi akT}{Gm^3}} \gg 1$$

and the term $g_{3/2}^{(\mu)}(1)$ allows us to reduce a bit overestimated predictions of standard BEC DM model.

$\mu-$BOSE GAS THERMODYNAMICS

We deform Bose gas thermodynamics using μ-calculus, i.e., in the relation between total number of particles and grand canonical partition function we replace usual derivative by μ-derivative:

$$N = z\frac{d}{dz}\ln Z \quad \rightarrow \quad N^{(\mu)} = z\mathcal{D}_z^{(\mu)} \ln Z.$$

As result we obtain thermodynamical functions of μ-Bose gas – total number of particles and grand canonical partition function (λ is thermal wavelength):

$$N^{(\mu)} = \frac{V}{\lambda^3} g_{3/2}^{(\mu)}(z) + g_0^{(\mu)}(z), \qquad \ln Z^{(\mu)} = \frac{V}{\lambda^3} g_{5/2}^{(\mu)} + g_1^{(\mu)}.$$

As we are interested in existence of phase transitions in considered system, we study this issue using thermodynamical geometry. Consider $2d$ space of thermodynamical parameters (β,γ) with $\beta = 1/k_B T$ and $\gamma = -\mu/k_B T$. Then, singularity of scalar curvature in this thermodynamical space signals presence of a phase transition in the system. Metric in the space is calculated as

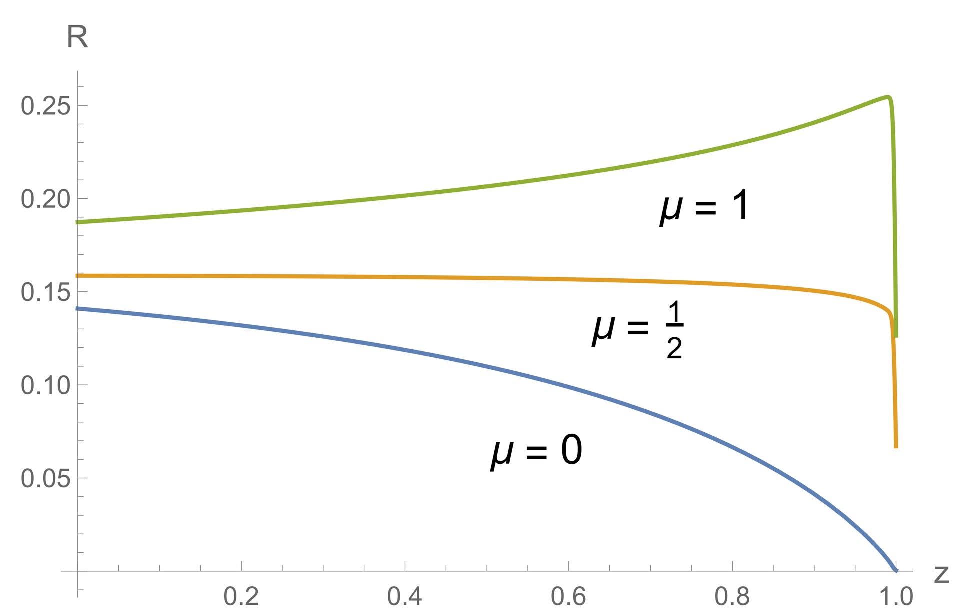
$$G_{\beta\beta} = \frac{\partial^2 \ln Z}{\partial\beta^2}, \qquad G_{\beta\gamma} = \frac{\partial^2 \ln Z}{\partial\beta^2}, \qquad G_{\gamma\gamma} = \frac{\partial^2 \ln Z}{\partial\beta^2}.$$

Then we find Christoffel symbols, Riemann tensor and scalar curvature according to the following equations

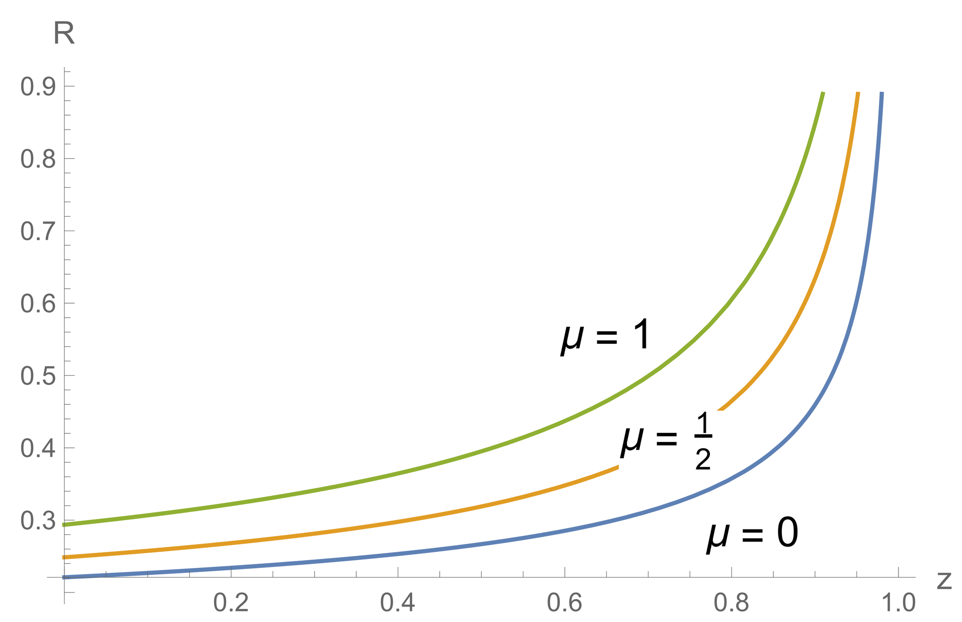
$$\Gamma_{\lambda\mu\nu} = \frac{1}{2}(\ln Z)_{,\lambda\mu\nu}, \quad R_{\lambda\mu\nu\rho} \equiv g^{\kappa\tau}\left(\Gamma_{\kappa\lambda\rho}\Gamma_{\tau\mu\nu} - \Gamma_{\kappa\lambda\nu}\Gamma_{\tau\mu\rho}\right), \quad R = \frac{2R_{1212}}{det|G|}.$$

The obtained geometric characteristics of thermodynamical space express through μ-polylogarithms (1) with $l = -2, -1, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$.

As seen in Fig. 2 below, phase transition (Bose-like condensation) in μ-Bose gas does exist for some values of thermodynamical parameters.



$V/\lambda^3 \sim 1$ and $V/\lambda^3 \ll 1$



$V/\lambda^3 \gg 1$

CRITICAL TEMPERATURE

Critical temperature of μ-Bose gas condensation can be inferred similarly to Bose gas case:

$$T_c^{(\mu)} = \frac{2\pi\hbar^2/mk}{\left(v g_{3/2}^{(\mu)}(1)\right)^{2/3}}.$$

For μ-deformed Bose gas, it is higher versus the ordinary one:

$$\frac{T_c^{(\mu)}}{T_c} = \left(\frac{2.61}{g_{3/2}^{(\mu)}(1)}\right)^{2/3} > 1.$$

The 2nd virial coefficient versus that of ordinary Bose gas is also to be mentioned, as it indicates presence of additional attraction between particles:

$$V_2^{(\mu)} - V_2^{Bose} = 2^{-5/2}\frac{\mu^2}{1+2/\mu} > 0$$

CONCLUSIONS

- The higher critical temperature provides wider range of temperatures admitting condensate state, and thus condensate state is more stable.
- Within our model it is possible to reduce predicted total mass of DM halo, relative to somewhat overestimated values in usual BEC DM model.
- The used μ-deformation implies an additional effective interaction, acting against the pressure.

REFERENCES

[1] T. Harko Bose-Einstein condensation of dark matter solves the core/cusp problem, JCAP 05 (2011) 022.

[2] A. Rebesh, A.M. Gavrilik, I. Kachurik Elements of μ-calculus and thermodynamics of μ-Bose gas model, Ukr. J. Phys. 85 (2013) 041123.

[3] A.M.Gavrilik, I.I.Kachurik, M.V.Khelashvili, A.V.Nazarenko, Condensate of μ-Bose gas as a model of dark matter, Physica A. 506 (2018) 835-843.