Corrections to Drell-Yan processes due to emission of extra lepton pair in PHOTOS and SANC

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Motivation

Recently it was published by ATLAS a precise measurement of $W$-boson mass $M_W^1$. Measurement of $M_W$ comes close with measurement of $Z$-boson mass $M_Z$

$$M_W^2 \left(1 - \frac{M_W^2}{M_Z^2}\right) = \frac{\pi \alpha}{\sqrt{2} G_\mu} (1 + \Delta r),$$

where $\alpha$ is fine structure, $G_\mu$ is Fermi constant, $\Delta r$ summarises the radiative corrections within the Standard Model.

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Introduction

There is convenient way to simulate the high-energy collisions:

- Parton shower Monte Carlo event generation (PYTHIA\textsuperscript{1} etc.)
- Calculation of the initial and final state radiation corrections (PHOTOS\textsuperscript{2}, SANC\textsuperscript{3})
- Detector response studies

The other way to obtain solution iteratively is not known, but such scheme is only an approximation of the theory!
As a result we have few different approximations!

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Hard process

On the right hand side is results of simulation of $pp \rightarrow Z \rightarrow e^+e^-$ by PYTHIA at 14 TeV center of mass energy.

$Z$ production and decay is represented by the Feynman graph on the left hand side.
Pair correction in PHOTOS and SANC

Real pair correction.

Virtual pair correction.
Pair correction in PHOTOS and SANC

On the right hand side is results of simulation of \( pp \rightarrow Z \rightarrow e^+ e^- \) by PYTHIA + PHOTOS

The correction to the born level process is presented on the left hand side. The correction is defined by

\[
\delta_{\text{pair}} = \left( \sigma_{\text{pair}} - \sigma_{\text{Born}} \right) / \sigma_{\text{Born}}
\]
Pair correction in PHOTOS

One can describe the amplitude of emission of extra lepton pair from final state, i.e. \( Z \rightarrow \ell^+ \ell^- + (f \bar{f}) \), like the Born amplitude multiplied by a factor\(^1\)

\[
\left| \frac{q}{q} \rightarrow \gamma/Z \ell^+ \ell^- + \frac{q}{q} \right|^2 = \left| \frac{q}{q} \rightarrow \gamma/Z \ell^+ \ell^- \right|^2 \times F(l^+,l^-,f,f)
\]

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Pair correction in PHOTOS

\[ \sum_{\text{spins}} |M_{f\bar{f}}|^2 = \frac{\alpha^4_{QED}(4\pi)^4}{(p_q + p_{\bar{q}})^4(p_f + p_{\bar{f}})^4} \]

\[ \times \text{Tr} [((p_{\bar{f}} + m_f)\gamma_{\alpha}(p_{\bar{f}} - m_f)\gamma_{\beta})] \]

\[ \times \text{Tr} [((p_{q} + m_f)\gamma^\nu(p_q - m_f)\gamma_{\mu})] \]

\[ \times \text{Tr} [(p_l- + m_l)] \]

\[ \times \left( \gamma^\alpha \frac{p_{l-} + p_f + p_{\bar{f}}}{(p_{l-} + p_f + p_{\bar{f}})^2 - m_l^2} \gamma^\mu \right. \]

\[ + \left. \gamma^\mu \frac{p_{l+} + p_f + p_{\bar{f}}}{(p_{l+} + p_f + p_{\bar{f}})^2 - m_l^2} \gamma^\alpha \right) \]

\[ \times (p_{l+} - m_l) \]

\[ \times \left( \gamma^\beta \frac{p_{l+} + p_f + p_{\bar{f}}}{(p_{l+} + p_f + p_{\bar{f}})^2 - m_l^2} \gamma^\nu \right. \]

\[ + \left. \gamma^\nu \frac{p_{l-} + p_f + p_{\bar{f}}}{(p_{l-} + p_f + p_{\bar{f}})^2 - m_l^2} \gamma^\beta \right) \]

\[ \times F(l^+, l^-, f, \bar{f}) \]

Pair correction
An important advantage of the described above procedure is that we can apply such factorization to a branch of any process that ends with decay to lepton pair

\[ \bar{q} \gamma/Z q + \bar{q} \gamma/Z f + \gamma/Z l^+ l^- f + \gamma/Z l^+ l^- f \]
Analytical verification of PHOTOS

- We first generate with PYTHIA the sample of events.
- In order to complete results for PHOTOS, its algorithm is applied on events generated by PYTHIA.
- For semi-analytical calculation we move events, that are generated by PYTHIA, to every possible bin of our test distributions with proper probabilities.

\[
\int F(l^+, l^-, f, f) \, d\Omega_4
\]

![Graph showing number of events vs. Me^+e^-[GeV]]
Analytical verification of PHOTOS

Corrections $\delta$ in % for invariant mass $M(e^+ e^-)$ distribution in $pp \rightarrow Z \rightarrow e^+ e^-$ at 14 TeV due to extra $e^+ e^-$ (left) or $\mu^+ \mu^-$ (right) pair emission.\(^1\)

Blue points represent semi-analytical results. Green dashed line represents SANC. Solid red line represents data by PYTHIA×PHOTOS.

Conclusions

- Agreement between pair implementation with the help of PHOTOS and SANC is insufficient, but the source of the PHOTOS-SANC differences is understood.

- Computation of distribution of the number of emissions of additional lepton-antilepton pair in the process $pp \rightarrow Z \rightarrow e^+e^-$ on the invariant mass of two electron-positron pairs by PHOTOS is in good agreement with such computation by analytical formula.

- Semi-analytical calculation is an extension of previous calculation\(^1\), where soft approximation for emission of pairs was used.

- Semi-analytical calculation supplements test of PHOTOS comparison to results for pair emission obtained from KORALW\(^2\) Monte Carlo for Z boson decay to 4 fermions, where extremely narrow width of intermediate Z boson is used to block emission of pairs from initial state.

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Thank you for your attention!
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Introduction

Pair correction

Backup slide

\[ F(l^+, l^-, f, \bar{f}) = -\frac{2}{3s} \left(\frac{\alpha}{\pi}\right)^2 \int dM_Q^2 \frac{dM_q^2}{M_q^2} \sqrt{1 - \frac{4\mu^2}{M_q^2}} \left(1 + \frac{2\mu^2}{M_q^2}\right) \left(\frac{m^2 \sqrt{1 - \frac{4m^2}{M_q^2}} \lambda^{\frac{1}{2}}(s, M_Q^2, M_q^2)}{M_q^2 M_Q^2 + \frac{m^2}{M_q^2} \lambda(s, M_Q^2, M_q^2)} + \frac{M_Q^2 - 2m^2}{s - M_q^2 - M_Q^2} \ln \frac{s - M_q^2 - M_Q^2 - \sqrt{1 - \frac{4m^2}{M_Q^2}} \lambda^{\frac{1}{2}}(s, M_Q^2, M_q^2)}{s - M_q^2 - M_Q^2 + \sqrt{1 - \frac{4m^2}{M_Q^2}} \lambda^{\frac{1}{2}}(s, M_Q^2, M_q^2)}\right) \]

where \( s = (p_q + p_{\bar{q}})^2 \), \( M_Q \) is invariant mass of the lepton pair, \( M_q \) is invariant mass of the additional lepton pair, \( m \) is mass of single lepton of first kind, \( \mu \) is mass of single additional lepton of second kind, \( \lambda = (s + M_q^2 - M_Q^2)^2 - 4sM_Q^2 \).
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Initialization parameters for PYTHIA.