2D Hawking radiation for massive scalar field theory

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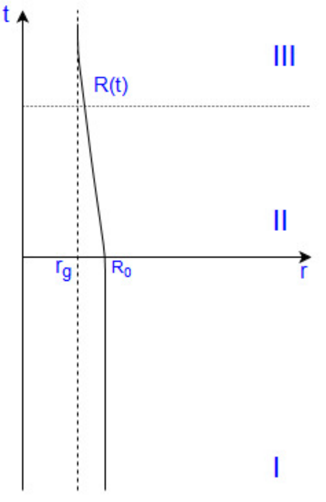
Introduction



Stages:

- I–before collapse, know modes exactly, no energy flux.

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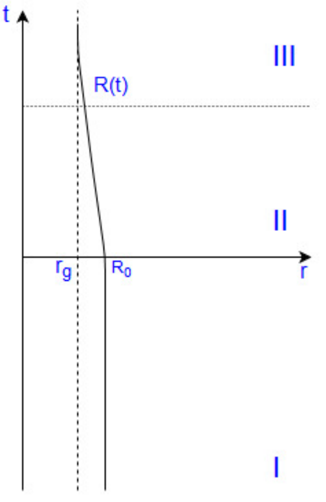

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- II–early stage of collapse, do not know modes.

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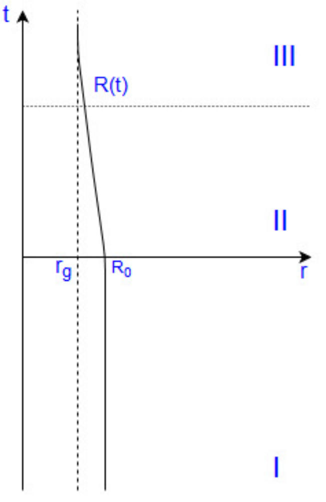

Stages:

- I–before collapse, know modes exactly, no energy flux.
- II–early stage of collapse, do not know modes.
- III–late stage of collapse, know modes approximately, non-zero Hawking flux.

Set up of the problem

We consider a massive scalar field on a thin-shell collapse background in 2D.

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4$$

Flat metric inside and Schwarzschild metric outside the shell

$$ds^2 = \begin{cases} dt_-^2 - dr^2, & r < R(t) \\ (1 - \frac{r_g}{r})dt^2 - \frac{dr^2}{1 - \frac{r_g}{r}}, & r > R(t) \end{cases}$$

Thin shell radius

$$R(t) = \begin{cases} R_0, & t \le 0 \\ R(t), & t > 0 \end{cases}$$

What is happening with massive scalar field during collapse?

Gravitational preliminary

$$ds_-^2 = ds_+^2 \Rightarrow t_- = t\sqrt{1 - \frac{r_g}{R_0}}, \quad t < 0$$

Shell trajectory

$$\mathrm{R(t)}{=}\mathrm{r}_g\left(1 + \frac{R_0 - r_g}{r_g} e^{-\frac{t}{r_g}}\right), \quad R(t_-) = R_0 - ct_-$$

The shell velocity:

$$c = \left|\frac{dR(t_-)}{dt_-}\right|$$

Time inside the shell

$$\mathrm{t}_- \approx \frac{R_0 - r_g}{c}\left(1 - e^{-\frac{t}{r_g}}\right), \quad t \to \infty.$$

Before the onset of collapse

Equation for the scalar field inside and outside, when r→ r_g

$$\begin{cases} \left(\partial_{t_-}^2 - \partial_r^2 + m^2\right)\phi = 0, & r < R_0, \\ \left(\partial_t^2 - \partial_{r_*}^2\right)\phi = 0, & r > R_0. \end{cases}$$

Boundary conditions

$$\begin{cases} \phi(0) = 0, \\ \phi(R_0 - 0) = \phi(R_0 + 0), \\ \dfrac{\partial t_-}{\partial t}\partial_r\phi(R_0 - 0) = (1 - \dfrac{r_g}{R_0})\partial_r\phi(R_0 + 0). \end{cases}$$

Scalar field before collapse

The massive mode

$$\phi = \begin{cases} Ae^{-i\omega_- t_-}\left(e^{-ikr} - e^{ikr}\right), & r < R_0, \\ e^{-i\omega t}\left(Be^{-i\omega r_*} + Ce^{i\omega r_*}\right), & r > R_0. \end{cases}$$

Using boundary conditions and that $\sqrt{1 - \frac{r_g}{R_0}} \to 0$ obtain:

The approximate massive mode

$$\phi \approx \begin{cases} \frac{e^{-1/2\omega\varphi}}{\sqrt{2\omega}} e^{-i\omega_- t_-}\left(e^{-ikr} - e^{ikr}\right), & r < R_0, \\ \frac{1}{\sqrt{2\omega}}\left(e^{-i\omega v} - e^{-i\omega(u+\varphi)}\right), & r > R_0. \end{cases}$$

Covariant point-splitting

Sress-energy tensor

$$\langle T_{\mu\nu}\rangle = \frac{1}{2}\langle \partial_\alpha \phi(x^+)\partial_\beta \phi(x^-) + \partial_\beta \phi(x^-)\partial_\alpha \phi(x^+)\rangle \Big(e_\mu^{+\alpha} e_\nu^{-\beta} - \\ - \frac{1}{2} g_{\mu\nu} g^{\sigma\rho} e_\sigma^{+\alpha} e_\rho^{-\beta} \Big) + \frac{m^2}{2} g_{\mu\nu} \frac{1}{2}\langle \phi(x^+)\phi(x^-) + \phi(x^-)\phi(x^+)\rangle,$$

where

$$x^\mu(\pm\epsilon) = x^\mu \pm \epsilon t^\mu + \frac{1}{2}\epsilon^2 a^\mu \pm \frac{1}{6}\epsilon^3 b^\mu,$$

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\nu\lambda}\frac{dx^\nu}{d\tau}\frac{dx^\lambda}{d\tau} = 0,$$

$$x^\mu(\tau = 0) = x^\mu, \qquad \frac{dx^\mu}{d\tau}(\tau = 0) = t^\mu = (t^u, t^v),$$

$$\frac{de^\mu_\nu}{d\tau} + \Gamma^\mu_{\rho\sigma}\frac{dx^\rho}{d\tau} e^\sigma_\nu = 0, \quad e^\mu_\nu(\tau = 0) = \delta^\mu_\nu$$

Stress-energy tensor

After direct computation

$$\langle T_{\mu\nu}\rangle = -\left[\frac{1}{4\pi\epsilon^2(t_\alpha t^\alpha)} - \frac{R}{24\pi}\right]\left[\frac{t_\mu t_\nu}{t_\alpha t^\alpha} - \frac{1}{2}g_{\mu\nu}\right] + \Theta_{\mu\nu} - \frac{m^2}{8\pi}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} - \frac{m^2}{16\pi}g_{\mu\nu}\log\left[4m^2\epsilon^2\frac{t_\alpha t^\alpha}{C}\right],$$

where

$$\Theta_{uu} = -\frac{1}{12\pi}C^{1/2}\partial_u^2 C^{-1/2},$$

$$\Theta_{vv} = -\frac{1}{12\pi}C^{1/2}\partial_v^2 C^{-1/2},$$

$$\Theta_{vu} = \Theta_{uv} = 0,$$

$$C = 1 - \frac{r_g}{r}.$$

Stress-energy tensor

Demand

$$\nabla^\mu \langle T_{\mu\nu} \rangle = 0,$$

then

Renormalized stress-energy tensor

$$\langle T_{\mu\nu} \rangle = \Theta_{\mu\nu} - \frac{R}{48\pi} g_{\mu\nu} - \frac{m^2}{8\pi} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

where

$$\Theta_{uu} = -\frac{1}{12\pi} C^{1/2} \partial_u^2 C^{-1/2},$$

$$\Theta_{vv} = -\frac{1}{12\pi} C^{1/2} \partial_v^2 C^{-1/2},$$

$$\Theta_{vu} = \Theta_{uv} = 0.$$

Stress-energy tensor

Components before collapse

$$\langle T_{uu} \rangle = \langle T_{vv} \rangle = \frac{\pi}{12} T_H^2 \Big[3(\frac{r_g}{r})^4 - 4(\frac{r_g}{r})^3 \Big] - \frac{m^2}{8\pi}, \quad T_H = \frac{1}{4\pi r_g}$$

In outside coordinates r, t the flux before the onset of collapse is

Flux

$$\langle T_t^r \rangle = \langle T_{uu} \rangle - \langle T_{vv} \rangle = 0$$

Long time after the collapse

Mode of the scalar field at t$\to\infty$

$$\phi_\omega \approx \begin{cases} \frac{e^{-i1/2\omega\varphi}e^{-i\omega_- t_-}}{\sqrt{2\omega}}\left(e^{-ikr}-e^{ikr}\right), & r<R(t), \\ \frac{1}{\sqrt{2\omega}}\left(e^{-i\omega(v+\phi_0)}+e^{-i\omega(BU+A)}\right), & |R(t)-r_g|<<r_g, \end{cases}$$

where

$$U = e^{-\frac{u}{2r_g}}\text{–Cruskal coordinate.}$$

Hence,

Stress-energy tensor components

$$\begin{cases} \langle T_{vv}\rangle = \frac{\pi}{12}T_H^2\left[3(\frac{r_g}{r})^4-4(\frac{r_g}{r})^3\right]-\frac{m^2}{8\pi} \\ \langle T_{uu}\rangle = \frac{\pi}{12}T_H^2\left(1-\frac{r_g}{r}\right)^2\left(1+2\frac{r_g}{r}+3(\frac{r_g}{r})^2\right) \end{cases}$$

Long time after the collapse

Flux long after the collapse

$$\langle T_t^r \rangle = \langle T_{uu} \rangle - \langle T_{vv} \rangle = \frac{\pi}{12} T_H^2 + \frac{m^2}{8\pi}$$

Then consider the self-interaction theory $\lambda \frac{\phi^4}{4!}$:

Self-interaction part in action for t$\rightarrow \infty$

$$\text{V(t)} = \frac{\lambda}{4!} \int_{R(t)}^{+\infty} \phi^4 dr + \frac{\lambda}{4!} \left(\frac{\partial t_-}{\partial t} \right) \int_{0}^{R(t)} \phi^4 dr, \quad \frac{\partial t_-}{\partial t} \approx e^{-\frac{t}{r_g}}.$$

Corrections to the Keldysh propagator

Contribution to the Keldysh propagator can be expressed as follows:

$$D^K(t_1,t_2) = \iint \frac{d\omega_1}{2\pi}\frac{d\omega_2}{2\pi}\Big\{\big[n_{\omega_1,\omega_2}+\delta(\omega_1-\omega_2)\big]\hat{\phi}^*_{\omega_1}(t_1,r_1)\hat{\phi}_{\omega_2}(t_2,r_2)+$$

$$\kappa_{\omega_1,\omega_2}\hat{\phi}_{\omega_1}(t_1,r_1)\hat{\phi}_{\omega_2}(t_2,r_2)+h.c.\Big\},$$

where

$$n_{\omega,\omega'}(t) = \frac{\lambda^2}{3}\int_{t_0}^{t}\int_{t_0}^{t} dt_3 dt_4 \int_{r_g}^{\infty}\int_{r_g}^{\infty} dr_3 dr_4 \hat{\phi}_\omega(t_3,r_3)\hat{\phi}^*_{\omega'}(t_4,r_4)$$

$$\prod_{i=1}^{3}\int_{m}^{+\infty}\frac{d\omega_i}{2\pi}\hat{\phi}_{\omega_i}(t_3,r_3)\hat{\phi}^*_{\omega_i}(t_4,r_4)$$

and almost the same for $\kappa_{\omega,\omega'}$. The point is that we obtain

$$n \approx \lambda^2 t, \quad \kappa \approx \lambda^2 t$$

Summary

- Hawking flux was obtained for the 2d thin shell collapse
- The corrections to the Keldysh propagator have the secular growth in the leading order

- Further studying:

Summarize all the leading corrections to the Keldysh propagator by the Dyson-Schwinger equation

Thanks for attention!