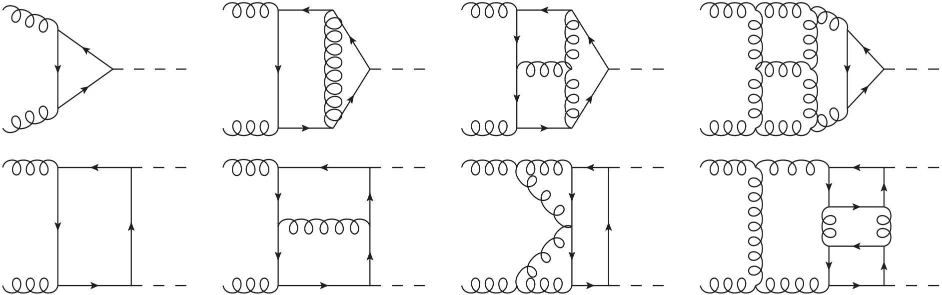
Karlsruhe Institute of Technology

Running, Decoupling & Higgs-Physics

Florian Herren | 15.06.2018

in collaboration with M. Gerlach, M. Steinhauser

INSTITUT FÜR THEORETISCHE TEILCHENPHYSIK



Higgs-Boson production in Gluon-fusion

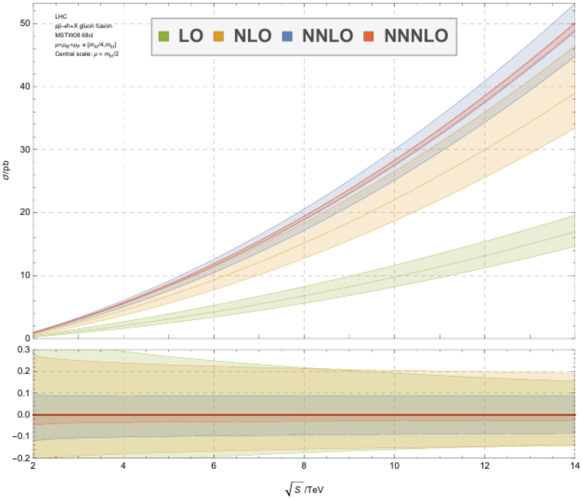
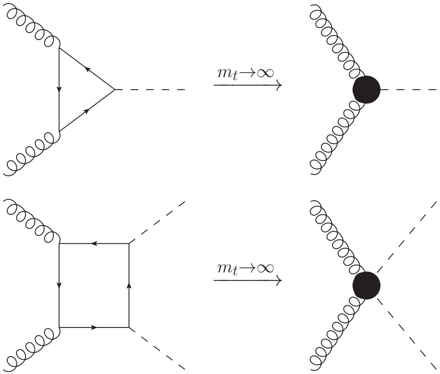


Figure: [Anastasiou et. al 2015]

Wilson coefficients for Higgs-Boson production



Wilson coefficients for Higgs-Boson production

- In the limit $m_t \to \infty$ the coupling of Higgs-bosons to gluons is given by

$$\mathcal{L}_{\mathrm{eff}} = -C_H \frac{H}{v} \mathcal{O}_1 + \frac{C_{HH}}{2} \left(\frac{H}{v}\right)^2 \mathcal{O}_1$$
$$\mathcal{O}_1 = G^a_{\mu\nu} G^{a\mu\nu}$$

- Inclusive Higgs-boson production known up to $\mathrm{N^3LO}$
  $\to C_H$ needed at 4 loops
- C_{HH} to 4 loops $\to \mathrm{N^3LO}$ Higgs pair-production

Interlude: Running & Decoupling in QCD

QCD beta function

QCD typically renormalized in $\overline{\mathrm{MS}}$-scheme $\to \alpha_s(\mu)$

$$\beta_{\alpha_s} = \mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} \frac{\alpha_s}{\pi} \qquad \beta_{\alpha_s} = -\sum_{i\geq 0} \beta_i \left(\frac{\alpha_s}{\pi}\right)^{i+2}$$

$$\beta_0 = \frac{1}{4}\left(11 - \frac{2}{3} n_f\right)$$

[Gross, Wilczek 1973], [Politzer 1973]

QCD beta function

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β_4 (5-loop) recently calculated:
[Baikov, Chetyrkin, Kühn 2016],
[Herzog, Ruijl, Ueda, Vermaseren 2017],
[Luthe, Maier, Marquard, Schroder 2017]

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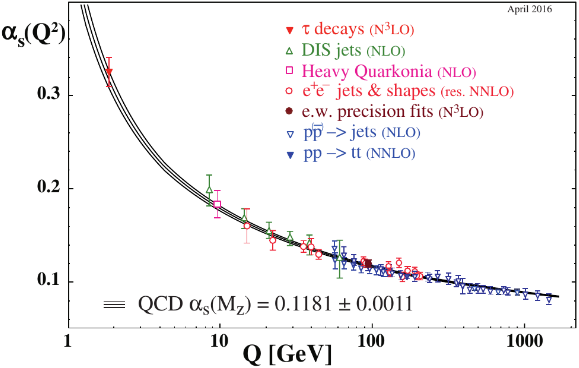

Figure: [PDG 2018]

Decoupling of heavy quarks

$\sqrt{s} \ll m_t \rightarrow$ work in effective 5-flavour theory
$\rightarrow$ decouple top-quark from running of α_s

$\overline{\mathrm{MS}}$ scheme:

$$\alpha_s^{(5)}(\mu_{\mathrm{dec}}) = \zeta_{\alpha_s}\left(\ln\left(\frac{\mu_{\mathrm{dec}}^2}{m_t^2}\right)\right)\alpha_s^{(6)}(\mu_{\mathrm{dec}})$$

ζ_{α_s} known to 4 loops for $N_C = 3$:
[Chetyrkin, Kühn, Sturm 2015], [Schroder, Steinhauser 2015]

Decoupling of heavy quarks

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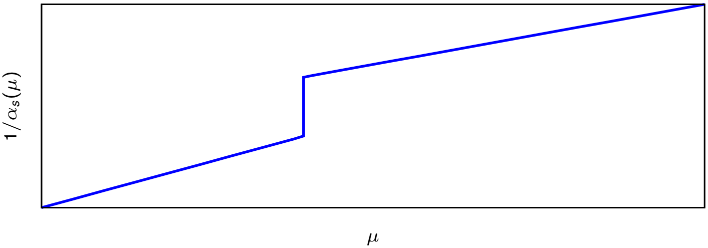
$\rightarrow \alpha_s(\mu)$ is not continuous:



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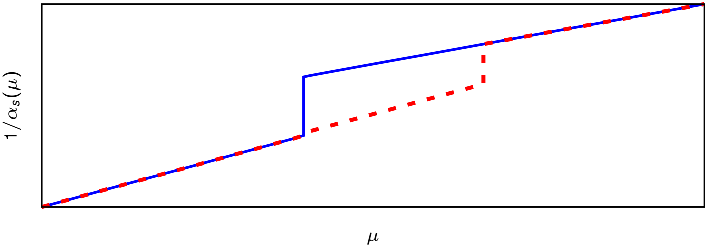


RunDec & CRunDec 3 [FH, Steinhauser, 2017]

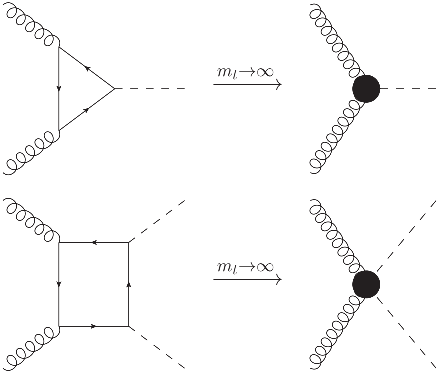
- QCD running & decoupling
- Relations between various mass-schemes
- highest perturbative accuracy available
- Mathematica and C++

RunDec: [Chetyrkin, Kühn, Steinhauser, 2000]

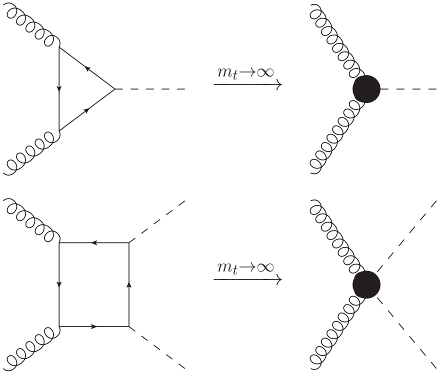
CRunDec: [Schmidt, Steinhauser, 2012]

Decoupling & Higgs-physics

$\mathcal{L}_{\text{eff}}$



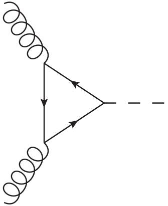
$\mathcal{L}_{\text{eff}}$



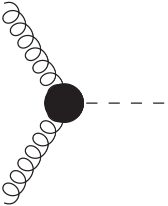
$$\mathcal{L}_{\text{eff}} = -C_H \frac{H}{v} \mathcal{O}_1 + \frac{C_{HH}}{2} \left(\frac{H}{v}\right)^2 \mathcal{O}_1$$

$$\mathcal{O}_1 = G^a_{\mu\nu} G^{a\mu\nu}$$

C_H



$$= \epsilon_1^\mu \epsilon_2^\mu \left(g_{\mu\nu}(q_1.q_2) - q_{1\,\nu} q_{2\,\mu}\right) F(m_t, m_H)$$

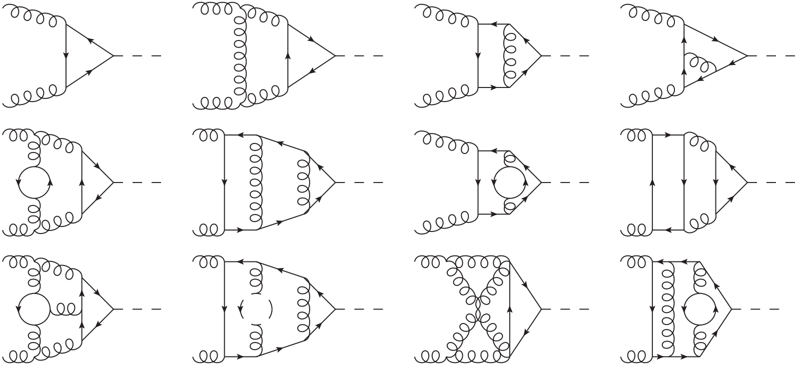


$$= \epsilon_1^\mu \epsilon_2^\mu \left(g_{\mu\nu}(q_1.q_2) - q_{1\,\nu} q_{2\,\mu}\right) \frac{C_H}{v}$$

$\rightarrow$ At leading order: $C_H = vF|_{m_t \rightarrow \infty}$

C_H

23251 diagrams:



C_H

- Expand F in $\frac{1}{m_t}$
- technical challenge: ”massive tadpoles” up to 4 loops

$$C_H = -\frac{\alpha_s^{(5)}}{3\pi}\left\{1 + ... + \left(\frac{\alpha_s^{(5)}}{\pi}\right)^3 \left[{C_A}^3\left(-\frac{110041}{124416} + \frac{1577}{9216}\zeta(3) - \frac{1993}{3456}\ln\frac{\mu^2}{m_t^2} - \frac{77}{576}\ln^2\frac{\mu^2}{m_t^2}\right) + ...\right]\right\}$$

Low-energy-theorem

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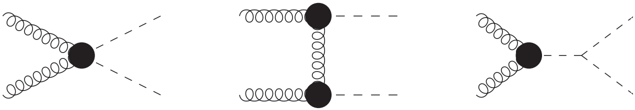
- Another way to compute C_H [Chetyrkin, Kniehl, Steinhauser 1997]:
- Low-Energy-Theorem (LET):

$$C_H = -\frac{2}{m_t^2 \zeta_{\alpha_s}} \frac{\partial}{\partial m_t^2} \zeta_{\alpha_s}$$

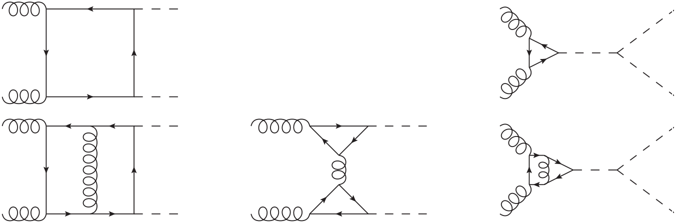
- Explicit calculation of C_H agrees with LET
- Good warmup for computing C_{HH}

C_{HH}

$$C_{HH}\mathcal{A}^{\text{eff}}_{1PI} + C_H^2\mathcal{A}^{\text{eff}}_{1PR,\lambda=0} + C_H\mathcal{A}^{\text{eff}}_{1PR,\lambda\neq0}$$



$$= \frac{1}{\zeta_3^0}\left(\mathcal{A}^{\text{full}}_{1PI} + \mathcal{A}^{\text{full}}_{1PR,\lambda=0} + \mathcal{A}^{\text{full}}_{1PR,\lambda\neq0}\right)$$



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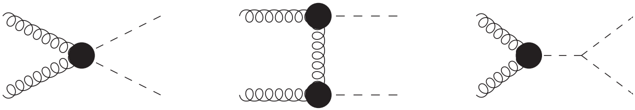

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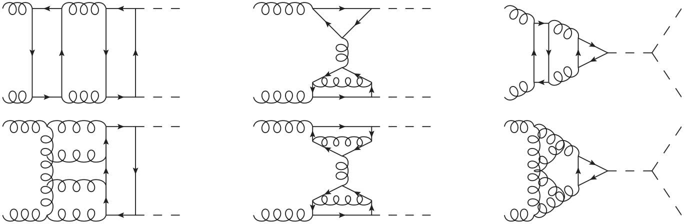

C_{HH}

- 145942 diagrams
- 1 and 2 loop $C_{HH} = C_H$
- 3 loops:

$$C_{HH} - C_H = \frac{7}{8}C_A^2 - \frac{5}{6}C_A T_F - \frac{11}{8}C_A C_F + \frac{1}{2}C_F T_F + C_F T_F n_l$$

[Grigo, Melnikov, Steinhauser 2014]

- At 4 loops there are 19 colour factors
- 16 of them completed
- 3 work in progress

$$C_{HH} - C_H = \left(\frac{\alpha_s^{(5)}}{\pi}\right)^4 \left[C_A^3\left(-\frac{1993}{1728} - \frac{77}{144}\ln\frac{\mu^2}{m_t^2}\right) + \ldots\right]$$

Low-energy-theorem

- [Spira 2016]:

$$C_{HH} = \frac{1}{4v^2}\left[\left(\frac{m_t}{\zeta_{\alpha_s}}\frac{\partial}{\partial m_t}\zeta_{\alpha_s}\right)^2 - \frac{m_t^2}{2\zeta_{\alpha_s}}\frac{\partial^2}{\partial m_t^2}\zeta_{\alpha_s}\right]$$

- The 16 colour factors of C_{HH} we explicitly computed agree

Summary

$$\mathcal{L}_{\text{eff}} = -C_H \frac{H}{v} \mathcal{O}_1 + \frac{C_{HH}}{2} \left(\frac{H}{v}\right)^2 \mathcal{O}_1$$

- Explicit calculation of C_H and C_{HH} at 4 loops
  $\rightarrow$ ingredient for $\mathrm{N}^3\mathrm{LO}$ production cross-sections
- Agreement with LET
- Byproduct: all decoupling constants in QCD at 4 loops