Higher spin particles

Maksim V. Khabarov

Moscow Institute of Physics And Technology /
Institute for High Energy Physics

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(Yu. M. Zinoviev, M.V. Khabarov; Nuclear Physics B, 2018)

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What is a higher spin particle

All known elementary particles have spin at most 1.

u	c	t	g	H
d	s	b	γ	
e	μ	τ	Z	
ν_e	ν_μ	ν_τ	W	

Higher spin particle is a particle with spin at least 3/2.

The need for higher spin particles theory

1. Physics beyond SM
2. Dynamics of compound particles in low-energy limit
3. Superstrings theory - infinite towers of higher spin particles

Interactions

Construction of gauge invariant interaction for arbitrary set of fields is impossible *(Berends, Burgers, van Dam; 1985)*.

The solution is to build an infinite tower of interacting particles in (A)dS space.

The representations of the Poincaré group

1. Massive particles (characterized by mass $p^2 = m^2 > 0$ and spin $s \in \mathbb{Z}/2$, $2s + 1$ polarizations)
2. Massless particles ($p^2 = 0$, characterized by helicity (spin) s, 2 different polarizations)
3. Infinite spin particles ($p^2 = 0$, characterized by two real parameters μ_0, μ_1, infinite number of polarizations)
4. Tachyons ($p^2 = m^2 < 0$)

(Anti)-de Sitter space

In curved spacetime $\partial_\mu \to D_\mu$, which is non-commutative.

$$[D_\mu, D_\nu]\xi_\alpha = R_{\mu\nu\alpha\beta}\xi^\beta \tag{1}$$

In constant curvature space $R_{\mu\nu\alpha\beta} = \kappa(g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha})$

1. $\kappa < 0$ - Anti-de Sitter space
2. $\kappa > 0$ - de Sitter space
3. $\kappa = 0$ - flat spacetime

Example: massless bosonic field

Massless field of spin s can be described with symmetric double-traceless tensor $\Phi^{\mu_1\mu_2\cdots\mu_s} = \Phi^{\mu(s)}$ with gauge transformation:

$$\delta\Phi^{\mu(s)} = \partial^\mu \xi^{\mu(s-1)} \tag{2}$$

with the gauge parameter $\xi^{\mu(s-1)}$ being a symmetric traceless tensor.

The lagrangian is

$$\begin{aligned}(-1)^s \mathcal{L}_0{}^{(s)} =& \frac{1}{2}\partial_\mu \Phi_{\beta(s)}\partial^\mu \Phi^{\beta(s)} - \frac{s}{2}(\partial\Phi)_{\beta(s-1)}(\partial\Phi)^{\beta(s-1)} \\ &+ \frac{s(s-1)}{2}\partial_\beta \tilde{\Phi}_{\beta(s-2)}(\partial\Phi)^{\beta(s-1)} + \ldots \end{aligned}\tag{3}$$

built up from the tensor $\Phi^{\mu(s)}$, its trace $\tilde{\Phi}^{\mu(s-2)}$ and their divergences. The gauge invariance vanishes either by introducing covariant derivatives or by adding mass terms. *(Singh, Hagen, 1974; Fang, Fronsdal, 1978)*

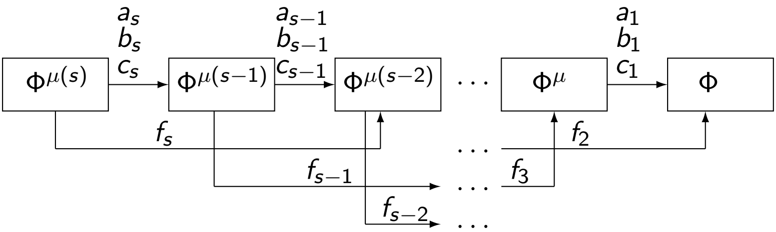
Example: massive bosonic field

Massive field of spin s in (A)dS can be described with a tower of $s+1$ massless fields with spins $0-s$ *(Zinoviev, 2001)*. The gauge transformations are changed and cross-terms and mass terms are added to the lagrangian:

$$\mathcal{L}^{(s)} = \sum_{k=0}^{s}(-1)^k \mathcal{L}_0{}^{(k)} + \sum_{k=1}^{s}(-1)^k \Big[\\ \left[a_k \Phi^{\beta(k)} D_\beta \Phi_{\beta(k-1)} + b_k \tilde{\Phi}^{\beta(k-2)} (D\Phi)_{\beta(k-2)} + c_k \tilde{\Phi}^{\beta(k-2)} D_\beta \tilde{\Phi}_{\beta(k-3)}\right] \\ + \left[d_k \Phi^{\beta(k)} \Phi_{\beta(k)} + e_k \tilde{\Phi}^{\beta(k-2)} \tilde{\Phi}_{\beta(k-2)} + f_k \tilde{\Phi}^{\beta(k-2)} \Phi_{\beta(k-2)}\right]\Big] \tag{4}$$

Field with spin k carries polarizations $\pm k$ (0 in case of scalar field) - $2s+1$ degrees of freedom in total. Construction of the fermion is similar.

Structure of the lagrangian



Unitarity and partially massless limits

Cross-coefficients $a_k^2, b_k^2, c_k^2, f_k^2$ are proportional to the term $[m^2 - \kappa(s-k)(s+k+d-5)]$, where m^2 is chosen to agree with the mass definition in flat space.

The unitarity condition for AdS and flat space is $m^2 \geq 0$, at $m = 0$ the field with the highest spin decouples.

In dS space $m^2 \geq s(s+d-5)\kappa$. In case $m^2 = (s-k)(s+k+d-5)\kappa$ non-unitary part decouples, and only polarizations $k+1, \ldots, s$ remain (**partially massless limit**).

Infinite spin

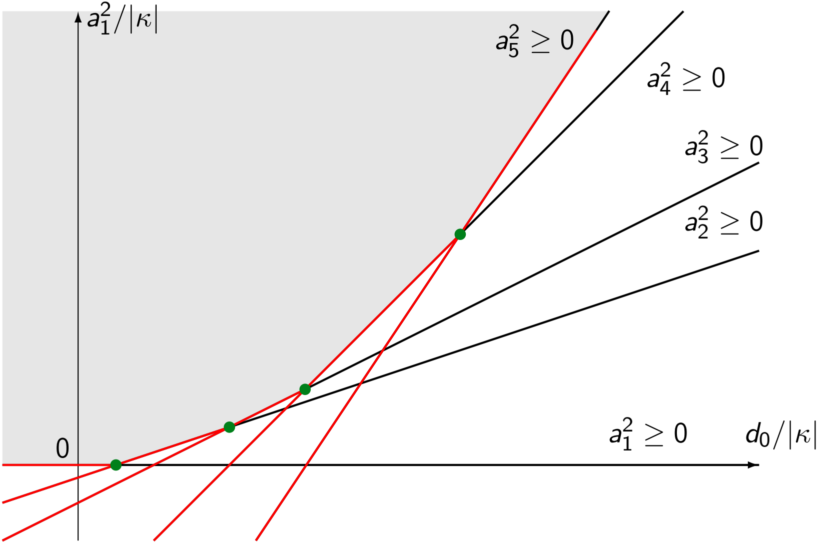
In case of $s = +\infty$, the lagrangian describes an infinite spin particle *(Metsaev, 2016)*. a_1 and d_0 can be chosen to describe the particle. The coefficients are proportional to
$\big[-\kappa x_k^2 + (\kappa(d-2) + 2d_0 - a_1^2)x_k + a_1^2(d-2)\big], x_k = (k-1)(d+k-4)$.

In flat space the unitarity condition is $2d_0 \geq a^2 \geq 0$, while the partially massless limits are given by

$$\begin{aligned} &2d_0 > a_1^2 \\ &2d_0(k-1)(d+k-4) = a_1^2(k-2)(d+k-3) \end{aligned} \tag{5}$$

In AdS the structure is more complicated...

Graphical interpretation of unitarity condition in AdS space



Frame-like formalism

A local orthonormal basis $e^{\mu}{}_{a}$ and dual basis $\hat{e}_{\mu}{}^{a}$ are introduced. Then all indices in $\Phi^{\mu(s)}$ except one are made local: $\Phi_{\mu}{}^{a(s-1)}$, and an auxiliary field $\Omega_{\mu}{}^{a(s),b}$ is introduced *(Vasiliev, 1980)*. The lagrangian for the massless spin-s field is

$$(-1)^s \mathcal{L} = \frac{1}{2}\hat{E}_{ab}\left[\Omega^{ac(k-2),d}\Omega^{b}{}_{c(k-2),d} + \frac{1}{k-1}\Omega^{c(k-1),a}\Omega_{c(k-1)}{}^{b}\right] + \hat{E}_{abc}\Omega^{a}{}_{d(k-2)}{}^{b}D\Phi^{cd(k-2)} \quad (6)$$

and the gauge transformations are:

$$\begin{aligned} \delta\Phi^{a(s-1)} &= \partial\xi^{a(s-1)} + e^{b}\eta^{a(s-1)}{}_{b} \\ \delta\Omega^{b(s-1),a} &= \partial\eta^{b(s-1),a} + e^{c}\zeta^{b(s-1),a}{}_{c} \end{aligned} \quad (7)$$

Frame-like formalism

Construction of massive field is similar to the metrical formalism.

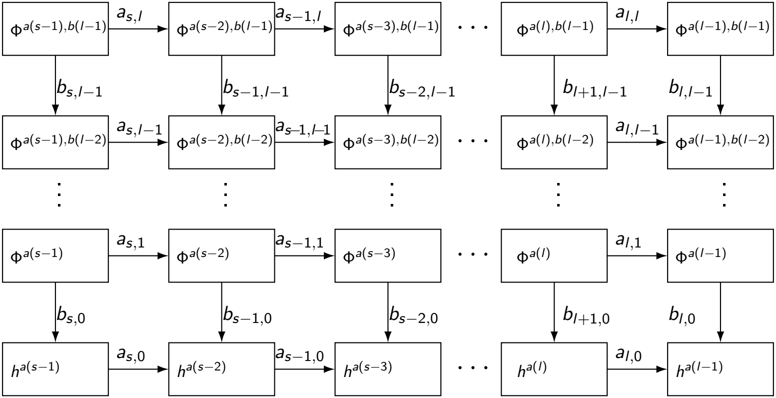
The physical particle is the same.

Higher dimensions

Symmetric tensors build up all irreducible representations for $d = 4$.

In $d = 5$, the tensors $\Phi^{a(s),b(l)}$, symmetric on $a(s)$ and $b(l)$ and such that $\Phi^{(a(s),a)b(l-1)} = 0$.

Structure of lagrangian



Current state of the work

1. Lagrangian for symmetric fermions in frame-like formalism (for arbitrary $d > 4$)
2. Lagrangians for symmetric infinite-spin particles in frame-like formalism
3. Lagrangians for mixed-symmetry (two-row Young diagram) infinite-spin particles in frame-like formalism

Plans for future

Build so-called unfolded equations for infinite spin fields (both bosons and fermions) in $d = 4$.

Thank you for your attention