Rôle of neutrino mixing in accelerated proton decay

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* M. Blasone, G. Lambiase, G. L., L. Petruzziello, Phys. Rev. D **97**, 105008 (2018)

Outline

Motivations

- Testing the consistency of QFT in curved background by comparing the decay rate of accelerated protons (*inverse β decay*) in the inertial and comoving frames: a ‘theoretical check" of the **Unruh effect**[*]

- Clarifying some conceptual issues concerning the inverse β decay in the context of neutrino mixing[†]

- Investigating the dichotomy between mass and flavor neutrinos as fundamental “objects" in QFT[‡]

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[*] G. E. A. Matsas and D. A. T. Vanzella, Phys. Rev. D **59**, 094004 (1999)
[†] D. V. Ahluwalia, L. Labun and G. Torrieri, Eur. Phys. J. A **52**, 189 (2016)
[‡] M. Blasone and G. Vitiello, Annals Phys. **244**, 283 (1995)

Unruh effect

- **Rindler coordinates**

$$x^0 = \xi \sinh\eta, \quad x^3 = \xi \cosh\eta$$

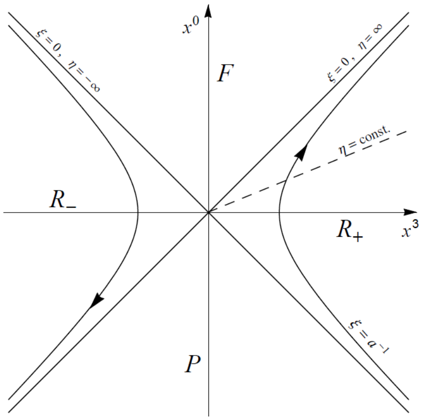
- Rindler and Minkowski metrics

$$ds^2_{\rm M} = (dx^0)^2 - (dx^3)^2 - (d\vec{x})^2 \rightarrow$$

$$ds^2_{\rm R} = \xi^2 d\eta^2 - d\xi^2 - (d\vec{x})^2$$

- Worldline of a Rindler observer

$$\eta = a\tau, \quad \xi = \mathrm{const} \equiv a^{-1}, \quad \vec{x} = \mathrm{const}$$



Fulling-Davies-Unruh effect

- Minkowski vacuum is a **thermal bath** for the Rindler observer

$$\langle 0_{\mathcal{M}}|\widehat{N}(\omega)|0_{\mathcal{M}}\rangle = \frac{1}{e^{a\omega/T_{\mathrm{U}}}+1}$$

- Unruh temperature

$$T_{\mathrm{U}} = \frac{a}{2\pi}$$

Neutrino mixing

Pontecorvo mixing transformations (two flavor model)...

$$|\nu_e\rangle = |\nu_1\rangle \cos\theta + |\nu_2\rangle \sin\theta$$

$$|\nu_\mu\rangle = -|\nu_1\rangle \sin\theta + |\nu_2\rangle \cos\theta$$

...lead to the quantum mechanical oscillation probability

$$P_{e\to\mu} = \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

Decay of accelerated particles

The decay properties of particles are less fundamental than commonly thought*.

$$\tau_{proton} > 10^{28} yrs.$$

* R. Muller, Phys. Rev. D **56**, 953 (1997)

Decay of accelerated particles

The decay properties of particles are less fundamental than commonly thought*.

$$\tau_{proton} > 10^{28} yrs.$$

However, in the presence of acceleration...

$$p \to n + e^{+} + \nu_e$$

...the proton decay is kinematically allowed!

Remark

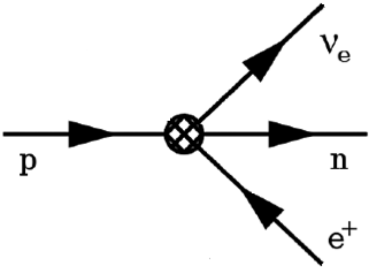
The lifetime of particles is not an absolute concept

* R. Muller, Phys. Rev. D **56**, 953 (1997)

Inverse β decay

Inertial frame

$$p \rightarrow n + e^+ + \nu_e$$

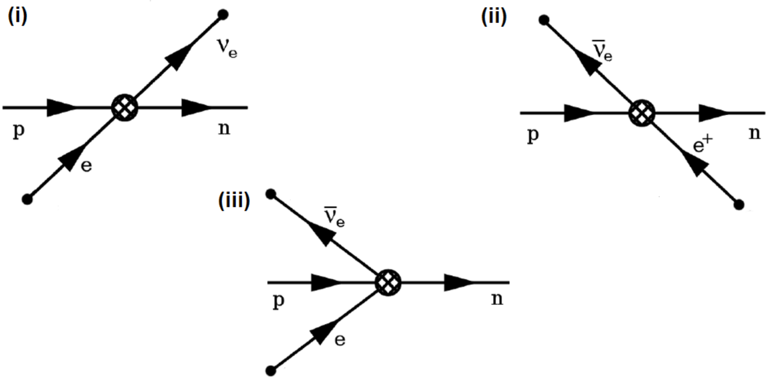


Inverse β decay

Comoving frame

$$(i)\ p + e \rightarrow n + \nu_e \qquad (ii)\ p + \bar{\nu}_e \rightarrow n + e^+$$

$$(iii)\ p + e + \bar{\nu}_e \rightarrow n$$



Setting the stage

In 2D with **massless** neutrino ($a \ll M_{Z^0}, M_{W^\pm} \approx 10^{36} cm/s^2$)*

$$\widehat{j}^\mu = \widehat{q}(\tau)\, u^\mu\, \delta\left(u - a^{-1}\right), \quad \widehat{q}(\tau) = e^{i\widehat{H}\tau}\, \widehat{q}_0\, e^{-i\widehat{H}\tau}$$

$$\widehat{H}\,|n\rangle = m_n\,|n\rangle\,, \quad \widehat{H}\,|p\rangle = m_p\,|p\rangle\,, \quad G_F = |\langle p|\,\hat{q}_0\,|n\rangle|$$

In this regime a **Fermi current-current interaction** can be considered

$$\widehat{S}_I = \int d^2x \sqrt{-g}\, \widehat{j}_\mu \left(\widehat{\overline{\Psi}}_\nu \gamma^\mu \widehat{\Psi}_e + \widehat{\overline{\Psi}}_e \gamma^\mu \widehat{\Psi}_\nu\right)$$

* G. E. A. Matsas and D. A. T. Vanzella, Phys. Rev. D **59** 094004 (1999)

Inertial frame calculation

Field quantization:

$$\widehat{\Psi} = \sum_{\sigma=\pm} \int_{-\infty}^{+\infty} dk \left[\widehat{b}_{k\sigma}\, \psi_{k\sigma}^{(+\omega)} + \widehat{d}_{k\sigma}^{\dagger}\, \psi_{-k-\sigma}^{(-\omega)} \right], \qquad \omega = \sqrt{m^2 + \mathbf{k}^2}$$

$$\psi_{k+}^{(\pm\omega)} = \frac{e^{i(\mp\omega x^0 + k x^3)}}{\sqrt{2\pi}} \begin{pmatrix} \pm\sqrt{(\omega \pm m)/2\omega} \\ 0 \\ k/\sqrt{2\omega(\omega \pm m)} \\ 0 \end{pmatrix}$$

$$\psi_{k-}^{(\pm\omega)} = \frac{e^{i(\mp\omega x^0 + k x^3)}}{\sqrt{2\pi}} \begin{pmatrix} 0 \\ \pm\sqrt{(\omega \pm m)/2\omega} \\ 0 \\ -k/\sqrt{2\omega(\omega \pm m)} \end{pmatrix}$$

Inertial frame calculation

The tree-level transition amplitude...

$$\mathcal{A}^{p\to n} = \langle n| \otimes \langle e^{+}_{k_e\,\sigma_e}, \nu_{k_\nu\,\sigma_\nu}| \widehat{S}_I\, |0\rangle \otimes |p\rangle$$

... and the related differential transition rate...

$$\frac{d^2\mathcal{P}^{p\to n}_{in}}{dk_e dk_\nu} = \sum_{\sigma_e=\pm}\sum_{\sigma_\nu=\pm} |\mathcal{A}^{p\to n}|^2\,, \qquad \frac{\mathcal{P}^{p\to n}}{T} = \Gamma^{p\to n}$$

... give the **inertial decay rate**

$$\Gamma^{p\to n}_{in} = \frac{4G_F^2 a}{\pi^2 e^{\pi\Delta m/a}} \int_0^\infty d\tilde{k}_e \int_0^\infty d\tilde{k}_\nu K_{2i\Delta m/a}\left[2\left(\tilde{\omega}_e + \tilde{\omega}_\nu\right)\right]$$

Comoving frame calculation

Field quantization:

$$\widehat{\Psi} = \sum_{\sigma=\pm} \int_0^{+\infty} d\omega \left[\widehat{b}_{\omega\sigma}\, \psi_{\omega\sigma} + \widehat{d}^{\dagger}_{\omega\sigma}\, \psi_{-\omega-\sigma}\right]$$

$$\psi_{\omega+} = \sqrt{\frac{m\cosh(\pi\omega/a)}{2\pi^2 a}} \begin{pmatrix} K_{i\omega/a+1/2}(m\xi) + iK_{i\omega/a-1/2}(m\xi) \\ 0 \\ -K_{i\omega/a+1/2}(m\xi) + iK_{i\omega/a-1/2}(m\xi) \\ 0 \end{pmatrix} e^{-i\omega\eta/a}$$

$$\psi_{\omega-} = \sqrt{\frac{m\cosh(\pi\omega/a)}{2\pi^2 a}} \begin{pmatrix} 0 \\ K_{i\omega/a+1/2}(m\xi) + iK_{i\omega/a-1/2}(m\xi) \\ 0 \\ K_{i\omega/a+1/2}(m\xi) - iK_{i\omega/a-1/2}(m\xi) \end{pmatrix} e^{-i\omega\eta/a}$$

Comoving frame calculation

The tree-level transition amplitude for each process...

$$\mathcal{A}^{p\to n}_{(\mathcal{I})} \;=\; \langle n| \otimes \langle\, emit\,|\widehat{S}_I|\, abs\,\rangle \otimes |p\rangle\,, \quad \mathcal{I} = i, ii, iii$$

... and the respective differential transition rates...

$$\frac{d^2\mathcal{P}^{p\to n}_{\mathcal{I}}}{d\omega_e d\omega_\nu} = \sum_{\sigma_e=\pm}\sum_{\sigma_\nu=\pm} \left|\mathcal{A}^{p\to n}_{\mathcal{I}}\right|^2 n_F^{(abs)}(\omega_{e(\nu)})[1 - n_F^{(emit)}(\omega_{\nu(e)})],$$

$$n_F(\omega) = \frac{1}{1 + e^{2\pi\omega/a}}$$

Comoving frame calculation

... give the **total (comoving) decay rate**

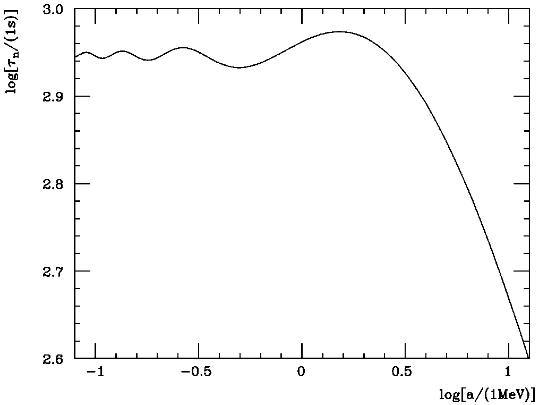
$$\Gamma^{p\to n}_{com} = \Gamma^{p\to n}_{(i)} + \Gamma^{p\to n}_{(ii)} + \Gamma^{p\to n}_{(iii)}$$

$$= \frac{G_F^2\, m_e}{a\,\pi^2\, e^{\pi\Delta m/a}} \int_{-\infty}^{+\infty} d\omega\, \frac{K_{i\omega/a+1/2}(m_e/a)K_{i\omega/a-1/2}(m_e/a)}{\cosh\left[\pi\left(\omega-\Delta m\right)/a\right]}.$$

Result

At tree level

$$\Gamma^{p\to n}_{in} = \Gamma^{p\to n}_{com}$$



Remarks

The equality of the two decay rates confirms:

- the necessity of Unruh effect in QFT
- the General Covariance of QFT in curved background

Generalizing to 4D with **massive neutrino***...

$$\widehat{j}^{\mu} = \widehat{q}(\tau) u^{\mu} \delta\left(u - a^{-1}\right) \delta(x^1)\delta(x^2), \quad \widehat{q}(\tau) = e^{i\widehat{H}\tau}\widehat{q}_0 e^{-i\widehat{H}\tau}$$

$$\widehat{H}\left|n\right\rangle = m_n \left|n\right\rangle, \quad \widehat{H}\left|p\right\rangle = m_p \left|p\right\rangle, \quad G_F = \left|\left\langle p\right| \hat{q}_0 \left|n\right\rangle\right|$$

$$\widehat{S}_I = \int d^4x \sqrt{-g}\, \widehat{j}_{\mu} \left(\widehat{\overline{\Psi}}_{\nu}\gamma^{\mu}\widehat{\Psi}_e + \widehat{\overline{\Psi}}_e \gamma^{\mu}\widehat{\Psi}_{\nu}\right)$$

* H. Suzuki and K. Yamada, Phys. Rev. D **67**, 065002 (2003)

Inertial frame calculation

Field quantization:

$$\widehat{\Psi} = \sum_{\sigma=\pm}\int d^3k\left[\widehat{b}_{\mathbf{k}\sigma}\psi^{(+\omega)}_{\mathbf{k}\sigma} + \widehat{d}^{\dagger}_{\mathbf{k}\sigma}\psi^{(-\omega)}_{-\mathbf{k}-\sigma}\right]$$

$$\psi^{(\pm\omega)}_{\mathbf{k}+}(x^0,\mathbf{x}) = \frac{e^{i(\mp\omega x^0+\mathbf{k}\cdot\mathbf{x})}}{2^2\pi^{\frac{3}{2}}}\frac{1}{\sqrt{\omega(\omega\pm m)}}\begin{pmatrix} m\pm\omega \\ 0 \\ k^3 \\ k^1+ik^2\end{pmatrix}$$

$$\psi^{(\pm\omega)}_{\mathbf{k}-}(x^0,\mathbf{x}) = \frac{e^{i(\mp\omega x^0+\mathbf{k}\cdot\mathbf{x})}}{2^2\pi^{\frac{3}{2}}}\frac{1}{\sqrt{\omega(\omega\pm m)}}\begin{pmatrix} 0 \\ m\pm\omega \\ k^1-ik^2 \\ -k^3\end{pmatrix}$$

Inertial frame calculation

Using the integral representation of the Bessel function

$$K_\mu(z) \;=\; \frac{1}{2}\int_{C_1}\frac{ds}{2\pi i}\Gamma(-s)\Gamma(-s-\mu)\left(\frac{z}{2}\right)^{2s+\mu}$$

together with the expansion formula...

$$(A+B)^z = \int_C \frac{dt}{2\pi i}\,\frac{\Gamma(-t)\,\Gamma(t-z)}{\Gamma(-z)}\,A^{t+z}\,B^t$$

Inertial frame calculation

...the decay rate in the inertial frame becomes

$$\Gamma_{in}^{p\to n} = \frac{a^5\, G_F^2}{2^5\, \pi^{7/2}\, e^{\Delta m/a}} \int_{C_s} \frac{ds}{2\pi i} \int_{C_t} \frac{dt}{2\pi i} \frac{\left(\frac{m_e}{a}\right)^2 \left(\frac{m_\nu}{a}\right)^2}{\Gamma(-s-t+3)\,\Gamma(-s-t+7/2)}$$

$$\times \Big[|\Gamma(-s-t+i\Delta m/a+3)|^2\, \Gamma(-s)\,\Gamma(-t)\,\Gamma(-s+2)\,\Gamma(-t+2)$$

$$+ \mathrm{Re}\Big\{ \Gamma(-s-t+i\Delta m/a+2)\,\Gamma(-s-t-i\Delta m/a+4) \Big\}$$

$$\times\, \Gamma(-s+1/2)\,\Gamma(-t+1/2)\,\Gamma(-s+3/2)\,\Gamma(-t+3/2)) \Big]$$

where $C_{s(t)}$ picks up all poles of gamma functions in $s(t)$ complex plane.

Comoving frame calculation

Field quantization:

$$\widehat{\Psi} = \sum_{\sigma=\pm} \int_0^{+\infty} d\omega \int d^2k \left[\widehat{b}_{\mathbf{w}\sigma} \psi^{(+\omega)}_{\mathbf{w}\sigma} + \hat{d}^{\dagger}_{\mathbf{w}\sigma} \psi^{(-\omega)}_{\mathbf{w}-\sigma}\right], \quad \mathbf{w} \equiv (\omega, k^x, k^y)$$

$$\psi^{(\omega)}_{\mathbf{w}+} = N \frac{e^{i(-\omega\eta/a + k_x x + k_y y)}}{(2\pi)^{\frac{3}{2}}} \begin{pmatrix} i l K_{i\omega/a-1/2}(\xi l) + m K_{i\omega/a+1/2}(\xi l) \\ -(k^1 + ik^2) K_{i\omega/a+1/2}(\xi l) \\ i l K_{i\omega/a-1/2}(\xi l) - m K_{i\omega/a+1/2}(\xi l) \\ -(k^1 + ik^2) K_{i\omega/a+1/2}(\xi l) \end{pmatrix}$$

with $l = \sqrt{m^2 + (k^x)^2 + (k^y)^2}$

Comoving frame calculation

Summing up the contributions of the three processes and using

$$x^\sigma K_\nu K_\mu = \frac{\sqrt{\pi}}{2} G^{40}_{24}\left(x^2 \middle| \begin{matrix} \frac{1}{2}\sigma,\ \frac{1}{2}\sigma + \frac{1}{2} \\ \frac{1}{2}(\nu+\mu+\sigma), \frac{1}{2}(\nu-\mu+\sigma), \frac{1}{2}(-\nu+\mu+\sigma), \frac{1}{2}(-\nu-\mu+\sigma) \end{matrix}\right)$$

the total decay rate in the comoving frame becomes

$$\Gamma^{p\to n}_{com} = \frac{a^5\, G_F^2}{2^5\, \pi^{7/2}\, e^{\Delta m/a}} \int_{C_s} \frac{ds}{2\pi i} \int_{C_t} \frac{dt}{2\pi i} \frac{\left(\frac{m_e}{a}\right)^2 \left(\frac{m_\nu}{a}\right)^2}{\Gamma(-s-t+3)\,\Gamma(-s-t+7/2)}$$

$$\times \Big[|\Gamma(-s-t+i\Delta m/a+3)|^2\, \Gamma(-s)\,\Gamma(-t)\,\Gamma(-s+2)\,\Gamma(-t+2)$$

$$+ \mathrm{Re}\Big\{\Gamma(-s-t+i\Delta m/a+2)\,\Gamma(-s-t-i\Delta m/a+4)\Big\}$$

$$\times\, \Gamma(-s+1/2)\,\Gamma(-t+1/2)\,\Gamma(-s+3/2)\,\Gamma(-t+3/2)\Big) \Big]$$

Proton decay and neutrino mixing: a theoretical paradox?

Recently, it has been argued that **neutrino mixing** can spoil the agreement between the two decay rates*

The *leitmotiv* is the violation of the KMS **thermal** condition for the accelerated neutrino vacuum. In particular, this would occur when one requires asymptotic (observed) neutrinos in the comoving frame to be in **flavor eigenstates**

The authors conclude by claiming that the contradiction must be solved **experimentally**

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* D. V. Ahluwalia, L. Labun and G. Torrieri, Eur. Phys. J. A **52**, 189 (2016)

~~The authors conclude by claiming that the contradiction must be solved **experimentally**~~

Remark

No experiment can be used for checking the internal consistency of theory against a theoretical paradox.

The question must be settled at a theoretical level, in conformity with the General Covariance of QFT in curved background.

The source of the ambiguity

Under the "magnifying glass"

Are the asymptotic neutrinos in mass or flavor eigenstates?

An attempt to solve the ambiguity has been proposed by requiring asymptotic neutrinos to be in **mass eigenstates** in both the inertial and comoving frames (on the basis that *flavor eigenstates make physical sense only for* $\delta m_{ij}^2 \sim 0)^*$...

Inverse β decay with mixed neutrinos

$$p \rightarrow n + \bar{\ell}_\alpha + \textcircled{\nu_i}, \qquad \ell = \{e, \tau, \mu\}, \quad i = \{1, 2, 3\}$$

* G. Cozzella *et al.* Phys. Rev. D **97**, 105022 (2018)

...but some criticism arises with reference to such a choice*:

- flavor eigenstates can be rigorously defined in any regime†
- if flavor states are well-defined for $\delta m_{ij}^2 \sim 0$, as those authors claim, why would they not use them, at least in that regime?
- using asymptotic mass neutrinos washes out mixing from calculations (and, indeed, in this case the same decay rates as in absence of mixing are obtained, up to a factor $\cos^2\theta$ or $\sin^2\theta$)

It is clear that some fundamental point is missing in the treatment of mixing by Cozzella *et al.*

* M. Blasone, G. Lambiase, G. L. and L. Petruzziello, arXiv:1804.11211 [hep-ph].
† M. Blasone and G. Vitiello, Annals Phys. **244**, 283 (1995)

Inertial frame calculation (our approach)

Requiring asymptotic neutrinos to be in **flavor eigenstates** as in Ahluwalia's approach*, the transition amplitude in the inertial frame becomes†

$$\mathcal{A}_{in}^{p\to n} = G_F\Big[\cos^2\theta\,\mathcal{I}_{\sigma_\nu\sigma_e}(\omega_{\nu_1},\omega_e) + \sin^2\theta\,\mathcal{I}_{\sigma_\nu\sigma_e}(\omega_{\nu_2},\omega_e)\Big]$$

$$\mathcal{I}_{\sigma_\nu\sigma_e}(\omega_{\nu_i},\omega_e) = \int_{-\infty}^{+\infty} d\tau\, e^{i\Delta m\tau}\, u_\mu\left[\bar{\psi}_{\sigma_\nu}^{(+\omega_{\nu_i})}\,\gamma^\mu\,\psi_{-\sigma_e}^{(-\omega_e)}\right], \qquad i=1,2$$

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* D. V. Ahluwalia, L. Labun and G. Torrieri, Eur. Phys. J. A **52**, 189 (2016)

† M. Blasone, G. Lambiase, G. L., L. Petruzziello, Phys. Rev. D **97**, 105008 (2018)

The decay rate thus takes the form

$$\Gamma_{in}^{p\to n} \;=\; \cos^4\theta\,\Gamma_1^{p\to n} + \sin^4\theta\,\Gamma_2^{p\to n} + \cos^2\theta\sin^2\theta\,\Gamma_{12}^{p\to n}$$

$$\Gamma_j^{p\to n} \;\equiv\; \frac{1}{T}\sum_{\sigma_\nu,\sigma_e} {G_F}^2 \int d^3k_\nu \int d^3k_e \left|\mathcal{I}_{\sigma_\nu\sigma_e}(\omega_{\nu_j},\omega_e)\right|^2, \quad j=1,2,$$

$$\Gamma_{12}^{p\to n} \;\equiv\; \frac{1}{T}\sum_{\sigma_\nu,\sigma_e} {G_F}^2 \int d^3k_\nu \int d^3k_e \Big[\mathcal{I}_{\sigma_\nu\sigma_e}(\omega_{\nu_1},\omega_e)\,\mathcal{I}^*_{\sigma_\nu\sigma_e}(\omega_{\nu_2},\omega_e) + \mathrm{c.c.}\Big]$$

Violating the KMS condition?

Assuming asymptotic neutrinos to be in **flavor eigenstates** *would violate the KMS definition of a thermal state of a quantum system by adding coherent, off-diagonal correlations in the density matrix. Consequently, the accelerated neutrino vacuum state would not be thermal, contradicting the essential characteristic of the Unruh effect**

* D. V. Ahluwalia, L. Labun and G. Torrieri, Eur. Phys. J. A **52**, 189 (2016)

Non-thermal Unruh effect for mixed neutrinos

Two Bogoliubov transformations involved*:

$$\phi_{\rm R} \xrightarrow{\text{thermal Bogol. } (a)} \phi_{\rm M} \;\Rightarrow\; \text{condensate in } |0_{\rm M}\rangle$$

$$\phi_1, \phi_2 \xrightarrow{\text{mixing Bogol. } (\theta)} \phi_e, \phi_\mu \;\Rightarrow\; \text{condensate in } |0_{e,\mu}\rangle$$

How do they combine when flavor mixing for an accelerated observer is considered?

* M. Blasone, G. Lambiase and G. G. Luciano, Phys. Rev. D **96**, 025023 (2017)

Non-thermal Unruh effect for mixed fields

Condensation density of **Rindler mixed neutrinos** in $|0\rangle_{\mathrm{M}}$:

$$\langle 0_{\mathrm{M}}|\widehat{N}(\theta,\omega)|0_{\mathrm{M}}\rangle = \underbrace{\frac{1}{e^{a\,\omega/T_{FDU}}+1}}_{Unruh\ thermal\ spectrum} + \underbrace{\sin^2\theta\left\{\mathrm{O}\left(\frac{\delta m^2}{m_{\nu_1}^2}\right)\right\}}_{non-thermal\ mixing\ corrections}$$

Remark

Non-thermal corrections only appear at orders higher than $\mathcal{O}\left(\frac{\delta m}{m}\right)$

Comoving frame calculation (Ahluwalia's approach)

Requiring asymptotic neutrinos to be in **mass eigenstates**, calculations in the comoving frame give for the process (*i*)

$$\mathcal{A}^{p\to n}_{(i)} \;=\; \frac{G_F}{a}\left[\cos\theta\,\mathcal{J}^{(1)}_{\sigma_\nu\sigma_e}(\omega_\nu,\omega_e)\;+\;\sin\theta\,\mathcal{J}^{(2)}_{\sigma_\nu\sigma_e}(\omega_\nu,\omega_e)\right]$$

$$\mathcal{J}_{\sigma_\nu\sigma_e} = \int_{-\infty}^{+\infty} d\eta\, e^{i\Delta m\,\eta}\, u_\mu\left[\bar{\psi}^{(\omega_\nu)}_{\mathbf{w}_\nu\sigma_\nu}\,\gamma^\mu\,\psi^{(\omega_e)}_{\mathbf{w}_e\sigma_e}\right]$$

Similar calculations for the other two processes lead to

$$\begin{aligned}\Gamma^{p\to n}_{com} \;&\equiv\; \Gamma^{p\to n}_{(i)} + \Gamma^{p\to n}_{(ii)} + \Gamma^{p\to n}_{(iii)}\\ &=\; \cos^2\theta\,\widetilde{\Gamma}^{p\to n}_{1} + \sin^2\theta\,\widetilde{\Gamma}^{p\to n}_{2}\end{aligned}$$

Comoving frame calculation (Ahluwalia's approach)

$$\Gamma^{p\to n}_{com} = \cos^2\theta\, \widetilde{\Gamma}^{p\to n}_1 + \sin^2\theta\, \widetilde{\Gamma}^{p\to n}_2$$

$$\widetilde{\Gamma}^{p\to n}_j = \frac{2\,G_F^2}{a^2\,\pi^7\,e^{\pi\Delta m/a}} \int_{-\infty}^{+\infty} d\omega \int d^2k_\nu\, l_{\nu_j} \left| K_{i(\omega-\Delta m)/a+1/2}\left(\frac{l_{\nu_j}}{a}\right)\right|^2$$

$$\times \int d^2k_e\, l_e \left| K_{i\omega/a+1/2}\left(\frac{l_e}{a}\right)\right|^2 + m_{\nu_j} m_e$$

$$\times\, \mathrm{Re}\left\{ \int d^2k_\nu K^2_{i(\omega-\Delta m)/a-1/2}\left(\frac{l_{\nu_j}}{a}\right) \int d^2k_e K^2_{i\omega/a+1/2}\left(\frac{l_e}{a}\right)\right\}$$

Comparing the rates

Inertial vs comoving rates

$$\Gamma_{in}^{p\to n} = \cos^4\theta\,\Gamma_1^{p\to n} + \sin^4\theta\,\Gamma_2^{p\to n} + \cos^2\theta\sin^2\theta\,\Gamma_{12}^{p\to n}$$

$$\Gamma_{com}^{p\to n} = \cos^2\theta\,\widetilde{\Gamma}_1^{p\to n} + \sin^2\theta\,\widetilde{\Gamma}_2^{p\to n}$$

Although:

$$\Gamma_j^{p\to n} = \widetilde{\Gamma}_j^{p\to n}, \quad j = 1,2$$

- there is no counterpart of the off-diagonal term in $\Gamma_{com}^{p\to n}$
- Pontecorvo matrix elements appear with different powers

Comoving frame calculation with flavor eigenstates (our approach)

Requiring asymptotic neutrinos to be in **flavor eigenstates**, calculations in the comoving frame now give for the process (i)

$$\mathcal{A}^{p\to n}_{(i)} = \frac{G_F}{a}\Big[\cos^2\theta\,\mathcal{J}^{(1)}_{\sigma_\nu\sigma_e}(\omega_\nu,\omega_e) + \sin^2\theta\,\mathcal{J}^{(2)}_{\sigma_\nu\sigma_e}(\omega_\nu,\omega_e)\Big],$$

$$\mathcal{J}_{\sigma_\nu\sigma_e}(\omega_\nu,\omega_e) = \int_{-\infty}^{+\infty} d\eta\, e^{i\Delta m\eta}\, u_\mu\big[\bar{\psi}^{(\omega_\nu)}_{\mathbf{w}_\nu\sigma_\nu}\,\gamma^\mu\,\psi^{(\omega_e)}_{\mathbf{w}_e\sigma_e}\big]$$

Analogous procedures for the other processes lead to

$$\begin{aligned}\Gamma^{p\to n}_{com} &\equiv \Gamma^{p\to n}_{(i)} + \Gamma^{p\to n}_{(ii)} + \Gamma^{p\to n}_{(iii)}\\ &= \cos^4\theta\,\widetilde{\Gamma}^{p\to n}_{1} + \sin^4\theta\,\widetilde{\Gamma}^{p\to n}_{2} + \cos^2\theta\sin^2\theta\,\widetilde{\Gamma}^{p\to n}_{12}\end{aligned}$$

$$\Gamma^{p\to n}_{com} = \cos^4\theta\,\widetilde{\Gamma}^{p\to n}_1 + \sin^4\theta\,\widetilde{\Gamma}^{p\to n}_2 + \cos^2\theta\sin^2\theta\,\widetilde{\Gamma}^{p\to n}_{12},$$

$$\begin{aligned}
\widetilde{\Gamma}^{p\to n}_{12} \;=\;& \frac{2\,G_F^2}{a^2\,\pi^7\,\sqrt{l_{\nu_1} l_{\nu_2}}\,e^{\pi\Delta m/a}}\int_{-\infty}^{+\infty} d\omega \left\{ \int d^2k_e\, l_e \left| K_{i\omega/a+1/2}\left(\frac{l_e}{a}\right)\right|^2 \right. \\
& \times \int d^2k_\nu \left(\kappa_\nu^2 + m_{\nu_1} m_{\nu_2} + l_{\nu_1} l_{\nu_2}\right) \\
& \times \mathrm{Re}\left\{ K_{i(\omega-\Delta m)/a+1/2}\left(\frac{l_{\nu_1}}{a}\right) K_{i(\omega-\Delta m)/a-1/2}\left(\frac{l_{\nu_2}}{a}\right)\right\} \\
& + m_e \int d^2k_e \int d^2k_\nu \left(l_{\nu_1} m_{\nu_2} + l_{\nu_2} m_{\nu_1}\right) \\
& \times \mathrm{Re}\left\{ K^2_{i\omega/a+1/2}\left(\frac{l_e}{a}\right) K_{i(\omega-\Delta m)/a-1/2}\left(\frac{l_{\nu_1}}{a}\right) \right. \\
& \left.\left. \times\; K_{i(\omega-\Delta m)/a-1/2}\left(\frac{l_{\nu_2}}{a}\right)\right\}\right\}, \quad \kappa_\nu \equiv (k_\nu^x, k_\nu^y)
\end{aligned}$$

Comparing the rates

Inertial vs comoving rates

$$\Gamma_{in}^{p\to n} = \cos^4\theta\,\Gamma_1^{p\to n} + \sin^4\theta\,\Gamma_2^{p\to n} + \cos^2\theta\sin^2\theta\,\Gamma_{12}^{p\to n},$$

$$\Gamma_{com}^{p\to n} = \cos^4\theta\,\widetilde{\Gamma}_1^{p\to n} + \sin^4\theta\,\widetilde{\Gamma}_2^{p\to n} + \cos^2\theta\sin^2\theta\,\widetilde{\Gamma}_{12}^{p\to n}$$

$$\Gamma_j^{p\to n} = \widetilde{\Gamma}_j^{p\to n}, \quad j = 1, 2$$

...what about the "off-diagonal" terms?

$$\Gamma_{12}^{p\to n} \stackrel{?}{=} \widetilde{\Gamma}_{12}^{p\to n}$$

Small neutrino mass difference

Evaluating these terms is non-trivial.

However, for $\frac{\delta m}{m_{\nu_1}} \equiv \frac{m_{\nu_2} - m_{\nu_1}}{m_{\nu_1}} \ll 1$,

$$\Gamma_{12}^{p\to n} = 2\Gamma_1^{p\to n} + \frac{\delta m}{m_{\nu_1}} \Gamma_{\delta_m} + \mathcal{O}\left(\frac{\delta m^2}{m_{\nu_1}^2}\right)$$

$$\widetilde{\Gamma}_{12}^{p\to n} = 2\widetilde{\Gamma}_1^{p\to n} + \frac{\delta m}{m_{\nu_1}} \widetilde{\Gamma}_{\delta_m} + \mathcal{O}\left(\frac{\delta m^2}{m_{\nu_1}^2}\right)$$

Result...

$$\frac{\Gamma_{\delta_m}}{m_{\nu_1}} = \frac{\widetilde{\Gamma}_{\delta_m}}{m_{\nu_1}}$$

... and its full expression

$$\begin{aligned}
\frac{\Gamma_{\delta_m}}{m_{\nu_1}} = & \lim_{\varepsilon \to 0} \frac{G_F^2\, m_e\, a^3}{\pi^3\, e^{\pi \Delta m / a}} \int_{C_s} \frac{ds}{2\pi i} \int_{C_t} \frac{dt}{2\pi i} \left(\frac{\varepsilon}{a}\right)^{2s+2} \left(\frac{m_e}{a}\right)^{2t+2} \\
& \times \frac{\Gamma(-2s)\Gamma(-2t)\Gamma(-t-1)\Gamma(-s-1)}{\Gamma(-s+\frac{1}{2})\Gamma(-t+\frac{1}{2})\Gamma(-2s-2t)} \\
& \times \left[\Gamma\left(-s-t+1+i\frac{\Delta m}{a}\right)\Gamma\left(-s-t-1-i\frac{\Delta m}{a}\right)\right. \\
& \left. + \Gamma\left(-s-t+1-i\frac{\Delta m}{a}\right)\Gamma\left(-s-t-1+i\frac{\Delta m}{a}\right)\right]
\end{aligned}$$

Result

Asymptotic neutrinos must be in **flavor eigenstates** in order to preserve the General Covariance of QFT in curved background

Overview

	Ahluwalia's approach	Cozzella's approach	Our approach
Asympt. neutrinos in the inertial frame	Flavor	Mass	Flavor
Asympt. neutrinos in the comoving frame	Mass	Mass	Flavor
Agreement	✗	✓ (incorrect treat-ment of mixing	✓ (in a suitable approximation)

Outlook

What happens beyond the first-order approximation?

$$\Gamma_{in}^{p\to n} = \Gamma_{com}^{p\to n} \quad \longrightarrow \quad \Gamma_{in}^{p\to n} \neq \Gamma_{com}^{p\to n}$$

$\Gamma_{in}^{p\to n} = \Gamma_{com}^{p\to n}$ $\downarrow$

The paradox would be solved at a theoretical level

$\Gamma_{in}^{p\to n} \neq \Gamma_{com}^{p\to n}$ $\downarrow$

- neutrino mixing is at odds with General Covariance
- Unruh effect with neutrino mixing should be revised
- Pontecorvo transformations are not consistent with QFT*

* M. Blasone and G. Vitiello Annals Phys. **244** 283 (1995)

Conclusions

- Unruh radiation gets non-trivially modified in the context of flavor mixing, losing its characteristic thermal behavior
- Neutrino asymptotic states must be **flavor eigenstates** in order to preserve the General Covariance of QFT
- The agreement between the two decay rates is restored, at least in the relativistic (first-order) approximation
- Work is in progress to establish what happens beyond the first-order approximation

THANKS FOR YOUR ATTENTION AND PLEASE ASK BUT NOT TOO MUCH