The effect of quantum fluctuations on compact star observables


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Motivation
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Quarks and Gluons

Quarkyonic phase?

Color Superconductor?

Net baryon density $n/n_0$ $n_0 = 0.16 \text{ fm}^{-3}$

Compact Stars

Transition

Proto-Neutron stars

NICA - MPD

FAIR SIS 300

RHIC

PHENIX

STAR

QCD
Motivation
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Hard to study area:

- no numerical results from lattice
- hard to reach with experiments

- Effective models
- indirect methods
- Astrophysics
- Compact stars
Model calculations for phase structure of nuclear matter shows the importance of correct treatment of quantum fluctuations in the bosonic sector, see: Kovacs, P. and Szep, Zs., PhysRevD.75.025015 (2007)
Model calculations for phase structure of nuclear matter shows the importance of correct treatment of **quantum fluctuations** in the **bosonic sector**, see: Kovacs, P. and Szep, Zs., PhysRevD.75.025015 (2007)

Quantum fluctuations may have important role in effective theories for **Cold Nuclear Matter**, which we can see in **Neutron Stars**.
Masquerade Problem:
Different EoS produce similar neutronstars

Advanced question:
Can the same Physical input (EoS) treated with different methods to calculate quantum fluctuations produce different neutron stars?

Method
FRG is a general, non-perturbative method to take quantum fluctuations into account, by solving the Wetterich-equation in different approximations.
FRG in practice: The Wetterich equation

- Exact equation for the scale dependence of the effective action, but it is very hard to solve directly
  - Scale dependent effective action ($k$ scale parameter)

$$\partial_k \Gamma_k = \frac{1}{2} \int dp^D \text{STr} \left[ \frac{\partial_k R_k}{\Gamma_k^{(2)}} + R_k \right]$$

$k=\Lambda$
Classical action

Integration

$k=0$
Quantum fluctuations included
Ansatz for the effective action:

\[ \Gamma_k [\varphi, \psi] = \int d^4x \left[ \bar{\psi} (i\slashed{\partial} - g\varphi) \psi + \frac{1}{2} (\partial_\mu \varphi)^2 - U_k (\varphi) \right] \]

**Fermions**: \( m=0, \) Yukawa–coupling generates mass

**Bosons**: the potential contains self interaction terms

We study the scale dependence of the potential only!!
Interacting Fermi–gas at finite temperature

Ansatz for the effective action:

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Wetterich –equation

\[ \partial_k U_k = \frac{k^4}{12 \pi^2} \left[ \frac{1 + 2 n_B (\omega_B)}{\omega_B} + 4 \frac{-1 + n_F (\omega_F - \mu) + n_F (\omega_F + \mu)}{\omega_F} \right] \]

Bosonic part

\[ U_\Lambda (\varphi) = \frac{m_0^2}{2} \varphi^2 + \frac{\lambda_0}{24} \varphi^4 \quad \omega_F^2 = k^2 + g^2 \varphi^2 \]

Fermionic part

\[ \omega_B^2 = k^2 + \partial_\varphi U \quad n_{B/F} (\omega) = \frac{1}{1 + e^{-\beta \omega}} \]
Integration of the Wetterich-equation

1.) Fix the high scale couplings in the theory

2.) Integrate the equation which is valid outside of the Fermi surface

3.) Calculate the initial conditions for the other equation inside the Fermi surface

4.) Integrate the equation which is valid below the Fermi surface
Results
Phase structure

![Graph showing phase structure with different approximations: First order, 1-loop approximation, Mean field approximation, and FRG-method. The graph plots the Yukawa coupling ($g_c(\lambda)$) against scalar self-interaction (x-axis).](image)
The equation of state

Pressure, $P [\text{MeV}^4]$ vs. Chemical potential, $\mu [\text{MeV}]$ for Yukawa-interacting Fermi gas EoS in MF approx., 1-Loop approx., and FRG-LPA.

PRESSURE

Mean Field $> 1$-LOOP $> \text{FRG}$
The equation of state
The compressibility

\[ \frac{1}{\chi} = n \frac{\partial p}{\partial n} \]
Application for compact stars

The Fermi-gas is not a realistic model of a neutron star, this is just demonstrates that an apparently small change in the EoS, due to quantum fluctuations means a noticeable change in the solution of the TOV equations.

\[
\begin{align*}
M_{\text{FRG}} &= 1.377 \\
M_{\text{MF}} &= 1.358 + 1.5\% \\
M_{1L} &= 1.309 - 3.5\%
\end{align*}
\]
The effect of quantum fluctuations

![Graph showing mass-radius relation and fluctuations](image)

- **Causality**
- **Rotation**

**Legend:**
- **MF**
- **Exact FRG-LPA**

**Axes:**
- **Mass [M_{\odot}]**
- **Radius [km]**

**Annotations:**
- \(\Delta R / R_{MF}\)
- \(\Delta M / M_{MF}\)

**Equation:**
\[
P_{21/19}
\]
Compactness

Mass [M_{\odot}]

FRG-LPA
1-Loop
MF
SQM3
WFF1

NICER predicted best accuracy
Thank you for the attention!

http://pospet.web.elte.hu/
(Contact and related materials)

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 Generating Functional+ Regulator
- The regulator acts as a **mass term** and suppresses fluctuations below scale $k$
- gradual momentum integration

$$Z_k[J] = \int \left( \prod_a d\Psi_a \right) e^{-S[\Psi] - \frac{1}{2} R_{k,ab} \Psi_a \Psi_b + \Psi_a J_a}$$

- The **effective action** is the Legendre–transform of the Schwinger functional:

$$\Gamma_k[\psi] = \sup_J (\psi_a J_a - W[J]) - \frac{1}{2} R_{k,ab} \psi_a \psi_b$$

- The scale–dependence of the effective action is given by the **Wetterich–equation**:

$$\partial_k \Gamma_k = \frac{1}{2} \text{Str} \left[ (\partial_k R_k) \left( \Gamma_k^{(1,1)} + R_k \right)^{-1} \right]$$
What does the ansatz exactly mean?

LPA is based on the assumption that the contribution of these two diagrams are close.

\textit{(momentum dependence of the vertices is suppressed)}

This implies the following ansatz for the effective action:

\[
\Gamma_k [\psi] = \int d^4 x \left[ \frac{1}{2} \psi_i K_{k,ij} \psi_j + U_k (\psi) \right]
\]
To use the original method we need an initial condition which do not have this mixing.

The boundary condition mix $k$ and $g\phi$.
We can transform the variables to make the quarter circle into a rectangle.

BUT now we have a well defined **boundary condition** too!
Interacting Fermi–gas at zero temperature

**Equations:**

\[ T=0, \mu \neq 0 \quad \Rightarrow \quad n_F(\omega) \rightarrow \Theta(-\omega) \]

We have two equations for the two values of the step function each valid on different domain.

**Graphical Representation:**

- **Fermionic vacuum fluctuations:**
  \[ \partial_k U_k = \frac{k^4}{12\pi^2} \left[ \frac{1}{\omega_B} - \frac{4}{\omega_F} \right] \]
  
  \[ k_F = \sqrt{\mu^2 - g^2 \varphi^2} \]

- **Thermodynamic fluctuations:**
  \[ \partial_k U_k = \frac{k^4}{12\pi^2} \frac{1}{\omega_B} \]

**Remark:**

Fermionic vacuum fluctuations and thermodynamic fluctuations cancel.
The solution changes only under the Fermi surface, because here we switch to the other equation.

Potential in one-loop approximation

Higher orders of the taylor-expansion for the square root converge fast where the potential is convex.
Why use renormalization in an effective theory?
Renormalization takes into account quantum fluctuations. This provides features one can not have in a mean field model.

What are these features?
- Quantum fluctuations play huge role in phase transitions – better description of phase transitions.
- FRG has a built in thermodynamical stability, which is not present in many mean field constructions; for example: Walecka model (the free energy is always convex)
- Better consistency with the quantum mechanic nature of the particles.
Frequently asked questions

- What is the meaning of FRG in an effective theory?
  It is a cutoff theory. It should provide a low energy effective description of QCD OR *Thinking in reverse: starting from low energy it could give us a hint of QCD at the cutoff: we can test what operators are important at that scale using the observations as constraints.*

- Is the effect of quantum fluctuations relevant in the case of compact stars?
  - It can change the neutron star mass for a given model, because the pressure of quantum fluctuations is taken into account
  - Possible new measurements (gravity waves) are more sensitive to the phase structure, which is better described using quantum fluctuations
  - Masquarade problem: many different model gives similar neutron star properties. Using FRG the quantum mechanical and thermodynamical consistency can help deciding between models.