Borelized version of QCD sum rules in the large-N_c limit

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Introduction

- Mesons play a big role in modern physics.
- E.g., π and σ mesons take part in nuclear physics.
- σ-meson (aka $f_0(500)$-meson) $\rightarrow$ the main part of the nucleon attraction potential.
- **Problem:** find meson masses!

Based on: https://arxiv.org/abs/1805.03089

Correlators

Masses can be found from studying the two-point correlators:

$$\Pi^J(Q^2) = \int d^4x\, e^{iQx} \langle \bar{q}\Gamma q(x) \bar{q}\Gamma q(0)\rangle_{planar},$$

$$\Gamma = i, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \; J = S, P, V, A$$

Quantum Chromodynamics (QCD) Sum Rules — from comparing two expansions of a correlator:

- Spectral representation
- Operator product expansion (OPE)

Correlators ⇒ Sum rules ⇒ Meson masses.

Borelized sum rules

- Improve accuracy of classic (planar) sum rules.
- Acquired by applying Borel transform:

$$L_M(Q^2) = \lim_{Q^2,k\to\infty;Q^2/k=M^2} \frac{1}{(k-1)!}(Q^2)^k \left(-\frac{d}{dQ^2}\right)^k$$

- Dispersion relation — a key equation:

$$\Pi(q^2) = \frac{1}{\pi}\int_{s_0}^{\infty} ds \frac{\operatorname{Im}\Pi(s)}{s-q^2-i\varepsilon}$$

- Borelized form:

$$L_M\Pi(q^2) = \frac{1}{\pi M^2}\int_{s_0}^{\infty} \exp(-s/M^2)\operatorname{Im}\Pi(s)ds$$

Spectral representation

Ansatz for spectral representation of correlator:

- Usual: "one infinitely narrow resonance + perturbative continuum".
- Works in large-N_c (**planar**) limit.
- Planar limit $\Rightarrow$ zero-width approximation + absence of multiparticle cuts.

M. A. Shifman, A. I. Vainshtein, V. I. Zakharov. "QCD and Resonance Physics. Theoretical Foundations". *Nucl. Phys.*, B147:385–447, 1979.

- **Ours:** "infinite set of infinitely narrow resonances"

$$\Pi(q^2) = \sum_n \frac{F^2}{q^2 - m_n^2 - i\varepsilon}$$

Linear Regge mass spectrum

Radial Regge trajectories:

$$m_n^2 = a \cdot n + m_0^2, \qquad n = 0, 1, 2, \ldots$$

- Dual models, hadron strings, etc.
- From logarithmic asymptotic behaviour of correlator:

$$\langle J(q)J(-q)\rangle = \sum_n \frac{F_n^2}{q^2 - M_n^2}$$

- Experimental observations

A. V. Anisovich, V. V. Anisovich, A. V. Sarantsev. "Systematics of $q\bar{q}$ states in the (n, M^2) and (J, M^2) planes." *Phys. Rev. D*, 62:051502, Jul 2000.

Vector sector

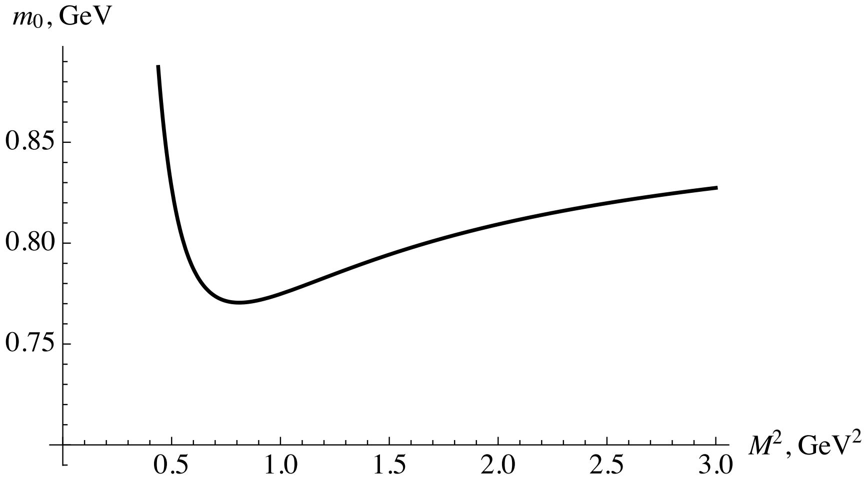
Correlators ⇒ Sum rules ⇒ Meson masses.

Solution:

$$m_0^2(M^2; a) = \frac{M^2\left[1 - \frac{h_2}{M^4} - \frac{2\cdot h_3}{M^6}\right]}{1 + \frac{h_1}{M^2} + \frac{h_2}{M^4} + \frac{h_3}{M^6}} - \frac{\mathbf{a}}{\mathbf{e^{a/M^2} - 1}}$$

- h_i — coefficients of borelized OPE
- Can be applied to other meson channels (ϕ, a_1) — just change h_i!
- Second term — correction to SVZ formula

"Borel window" for vector mesons



Mass spectra: "vector" cases

ρ-meson; $a = 1.5195 \pm 0.0695\,\text{GeV}^2$:

n	0	1	2	3
Comp.	770 ± 10	1450 ± 20	1910 ± 40	2230 ± 50
Exp.	770 ± 10	1465 ± 25	$1909 \pm 17 \pm 25$	2265 ± 40

a_1-meson; $a = 1.3 \pm 0.184\,\text{GeV}^2$:

n	0	1	2	3
Comp.	1150 ± 40	1620 ± 60	1980 ± 90	2280 ± 120
Exp.	1230 ± 40	1647 ± 22	1930^{+30}_{-70}	2270^{+55}_{-40}

Scalar sector

Two solutions:

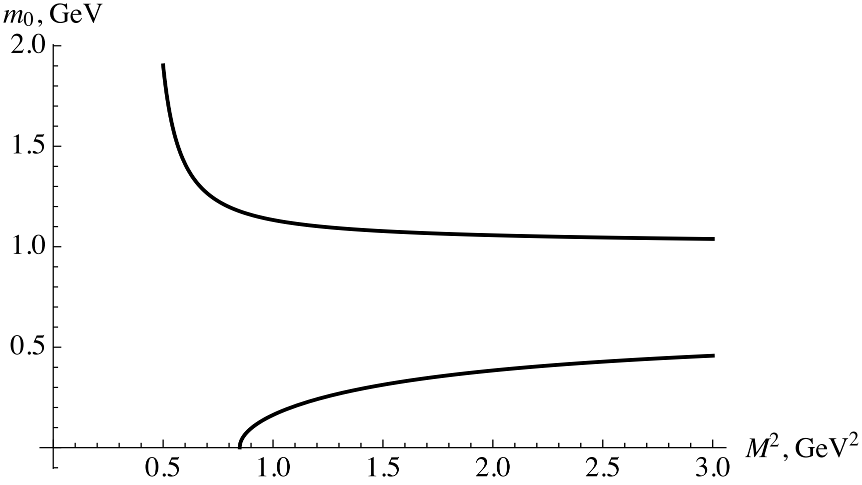
$$m_0^2 = \frac{1}{2}\left[\frac{2a}{1-e^{a/M^2}} + \mathbf{L} - \frac{\sqrt{-4a^2 e^{a/M^2} + \mathbf{L}^2\left(e^{a/M^2}-1\right)^2}}{e^{a/M^2}-1}\right]$$

$$m_0^2 = \frac{1}{2}\left[\frac{2a}{1-e^{a/M^2}} + \mathbf{L} + \frac{\sqrt{-4a^2 e^{a/M^2} + \mathbf{L}^2\left(e^{a/M^2}-1\right)^2}}{e^{a/M^2}-1}\right]$$

Reason — increased dimension of correlator (masses in residues):

$$\Pi(q^2) = \sum_n \frac{F^2\, m_n^2}{q^2 - m_n^2 - i\varepsilon}.$$

"Borel window" for scalar mesons?



Mass spectra: scalar case

f_0-meson; $a = 1.384 \pm 0.069\,\text{GeV}^2$

n	0	1	2	3
Comp.	1000 ± 30	1540 ± 20	1940 ± 40	2270 ± 50
Exp.	990 ± 20	1504 ± 6	1992 ± 16	2189 ± 13

n	0	1	2	3
Comp.	620	1330 ± 30	1780 ± 40	2130 ± 50
Exp.	400–550	1200–1500	1723^{+6}_{-5}	2101 ± 7

Conclusions

- Borelized planar sum rules $\Rightarrow$ mass spectra $\approx$ exp. data.
- Two solutions for scalar mesons!
- Second trajectory $\rightarrow f_0(500)$-meson $+$ other f_0 states.
- **Bonus:** No analogue for scalar sum rule in non-borelized planar sum rules.