Mixed QCD-EW corrections to vector-boson production

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QCD computations have reached N3LO

Fully-differential description of color-singlet production at the LHC has recently reached N3LO in perturbative QCD.

⇒ Higgs production [Chen et al. '21]
⇒ Drell-Yan [Chen et al. '22]

Parametrically, $\alpha_s^3 \sim \alpha_s \alpha$.

$\alpha_s(M_Z) \sim 0.1$, $\alpha \sim 1/100$

These computations are vital for precise studies at the LHC.
$M_W$ as a precision test

electroweak fits

\[ M_W^2 (1 - M_W^2/M_Z^2) = \frac{\pi \alpha}{\sqrt{2} G_F} (1 + \Delta r) \]

$M_W = 80354 \pm 7$ MeV [Gfitter’18]
$M_W = 80359 \pm 5$ MeV [HEPfit’21]

recent direct measurements

[ATLAS ’17] : 80370 ± 7 stat ± 11 sys ± 14 mod MeV
[LHCb ’21] : 80354 ± 23 stat ± 10 exp ± 17 th ± 9 pdf MeV
[CDF ’22] : 80433.5 ± 6.4 stat ± 6.9 sys MeV

comparison
Measurement of $M_W$ (at hadron colliders)

- through leptonic decay $pp \rightarrow W^* \rightarrow \ell \bar{\nu}$
- partonic center-of-mass energy $\hat{s}$ not known, neutrino momentum $p_\nu$ cannot be measured
  \[ \Rightarrow \text{measurements use shape of distributions in } p_\ell^\perp \text{ and } m_W^\perp \]

- $m_W^\perp = \sqrt{2p_\ell^\perp p_{\text{miss}}^\perp (1 - \cos \phi_{\ell,\text{miss}})} \leq M_W$
- $W$-boson width, pile-up
- see [Smith,van Neerven,Vermaseren '83]

- at LO: $p_\ell^\perp \leq m_W/2$
- perturbative corrections

[Calame et al. '16]
Measurement of $M_W$ (at hadron colliders)

template fit

collinear factorization [Collins, Soper, Sterman '88]

$$\sigma = \sum_{ij} \int_0^1 dx_1 dx_2 f_i(x_1) f_j(x_2) \sigma_{ij}(x_1, x_2) \left(1\% \right) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{Q^2}\right)$$

$\Rightarrow p_W^\perp$ spectrum can not be predicted to required precision.

$8/80'000 = 0.01\%$

- experimental analyses use $Z$-boson data to predict $p_W^\perp$ distribution
- hence, measurement is sensitive to effects that distinguish between $W$ and $Z$
- these include PDFs, massive-quark effects, NLO EW and NNLO QCD-EW corrections
Vector-boson production at $\mathcal{O}(\alpha_s \alpha)$

- $p^W_Z/p^Z_\perp$ distribution is dominated by the on-shell production of vector bosons

\[
\frac{d\sigma}{\alpha_s \alpha} = \left| \begin{array}{c} \text{production} \times \text{decay} \\ \alpha s \end{array} \right|^2 + \left| \begin{array}{c} \text{production} \\ \alpha s \alpha \end{array} \right|^2 + \mathcal{O}\left( \frac{\Gamma_V}{M_V} \right)
\]

[Dittmaier et al. '14 '16] [Behring et al. '19 '20]

- full, off-shell process $pp \rightarrow \ell \bar{\ell}$ was completed recently [Bonciani et al. '21] [Bucioni et al. '22]
Infrared divergences at NLO

\[ \left. \frac{d \sigma}{d \mathcal{O}} \right|_{\alpha_s} = 2 \Re \left[ \hat{\mathcal{O}} \Phi d \Phi X + \mathcal{F}_1 \Phi + g + \sigma_{pdf} \right] \]

\[ \sim \frac{1}{\varepsilon^2} \]

- cross-section is \textit{finite} for arbitrary “infrared-safe” observable \( \mathcal{F} \)
Infrared divergences at NLO

\[ \left. \frac{d\sigma}{dO} \right|_{\alpha_s} = 2 \Re \left[ \begin{array}{c} \vcenter{\hbox{\includegraphics[width=1cm]{diagram1}}} \\ \sim 1/\epsilon^2 \end{array} \right] \mathcal{F}_O d\Phi_X + \mathcal{F}_O^{(1)} d\Phi_{X+g} + d\sigma^{gq} + d\sigma^{\text{pdf}} \]

- cross-section is \textit{finite} for arbitrary “infrared-safe” observable \( \mathcal{F} \)
- individually, soft and collinear divergences arise from
Infrared divergences at NLO

\[
\frac{d\sigma}{dO}\bigg|_{\alpha_s} = 2\Re \left[ \left( \frac{p_1}{p_2} \right) \times \left( \frac{p_3}{p_4} \right) \right] \mathcal{F}_O d\Phi_X + \left[ \left( \frac{p_1}{p_5} \right) \times \left( \frac{p_3}{p_4} \right) \right] \mathcal{F}_O^{(1)} d\Phi_{X+g} + d\sigma^{gq} + d\sigma^{pdf}
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- cross-section is \textit{finite} for arbitrary “infrared-safe” observable \( \mathcal{F} \)
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  - \textit{loop integrals} in virtual corrections
Infrared divergences at NLO

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- cross-section is **finite** for arbitrary “infrared-safe” observable \( \mathcal{F} \)
- individually, soft and collinear divergences arise from
  - **loop integrals** in virtual corrections
  - **phase-space integration** over on-shell momenta of final-state partons in real-emission corrections

\[
\frac{1}{p_1 \cdot k_4} \sim \frac{1}{E_1 E_4 (1 - \cos \theta_{14})} \]

\[ \sim 1/\epsilon^2 \]

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Infrared divergences at NLO

\[
\frac{d\sigma}{dO} \bigg|_{\alpha_s} = 2 \Re \left[ \Phi_0 d\Phi_X + \Phi_1^{(1)} d\Phi_{X+g} + d\sigma^{gq} + d\sigma^{\text{pdf}} \right] \sim 1/\varepsilon^2
\]

- cross-section is *finite* for arbitrary “infrared-safe” observable \( \Phi \)
- individually, soft and collinear divergences arise from
  - loop integrals in virtual corrections
  - phase-space integration over on-shell momenta of final-state partons in real-emission corrections
- need to be regulated, extracted and cancelled
Infrared divergences at NLO

\[ \frac{d\sigma}{dO} \bigg|_{\alpha_s} = 2 \Re \left[ \mathcal{F}_0 d\Phi_X + \mathcal{F}_0^{(1)} d\Phi_{X+g} + d\sigma^{gq} + d\sigma^{pdf} \right] \]

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Infrared divergences at NLO

\[ \frac{d\sigma}{dO} \bigg|_{\alpha_s} = 2 \Re \left[ \frac{1}{p_1 \cdot k_4} \sim \frac{1}{E_1 E_4 (1 - \cos \theta_{14})} \right] \]

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- need to be regulated, extracted and cancelled
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\[ \Rightarrow \text{slicing and subtraction schemes} \]

are used to define contributions that are individually finite

\[ d\sigma \bigg|_{\alpha_s} = d\sigma_{X+1} + d\sigma_X \]
Infrared subtraction at NLO

- **general idea** is to subtract and add back matrix elements that approximate singular behavior

\[
d\sigma^{q\bar{q}}(d = 4 - 2\varepsilon) = \left(\frac{d\sigma^- - d\tilde{\sigma}^-}{d = 4}\right) + \frac{d\tilde{\sigma}^r(\varepsilon)}{}
\]

- **pole cancellation**

\[
\lim_{\varepsilon \to 0} \left[ d\sigma^{pdf} + d\sigma^V(\varepsilon) + d\tilde{\sigma}^r(\varepsilon) \right] = \text{finite}
\]

- **example** for soft-regulated emission of a gluon

\[
d\sigma^{q\bar{q}} = \left\{ \begin{array}{c} \begin{array}{c} \frac{1}{2} \left( \frac{2g_s^2C_F}{(p_1 \cdot k_4)(p_2 \cdot k_4)} \right) + 2 \left( \int [dk_4] \frac{2g_s^2C_F(p_1 \cdot p_2)}{(p_1 \cdot k_4)(p_2 \cdot k_4)} \right) \end{array} \\
\end{array} \right\}
\]

- **NB** similar formula for remaining collinear divergence
Quark emission at NNLO QCD-EW

\[ |\mathcal{A}_{ud \rightarrow W^+dd}|^2 \rho(\alpha_s \alpha) \]

- only triple-collinear divergence \( p_2 \parallel k_4 \parallel k_5 \) due to continuous quark line

\[
d\sigma_{rud \rightarrow W^+dd}^{rr} = \langle [dk_4][dk_5] (I - \hat{C}_2) F_{LM}(1_u, 2_d, W^+; 4_d, 5_d) \rangle \hspace{1cm} \leftarrow \text{fully regulated}
\]

\[
+ \langle [dk_4][dk_5] \hat{C}_2 F_{LM}(1_u, 2_d, W^+; 4_d, 5_d) \rangle \hspace{1cm} \leftarrow \text{subtraction term}
\]

- integrated triple-collinear subtraction terms were computed in the context of NNLO QCD corrections [MD,Melnikov ‘19], results can be re-used here

\[
\sim \alpha_s \alpha \times \int_0^1 \text{dz} \left[ f_1(z) \underbrace{\frac{1}{\varepsilon}}_{\text{pole cancellation}} + f_0(z) \right] \left\langle \frac{C_F Q_d^2 F_{LM}(1_u, z \cdot 2_d)}{z} \right\rangle, \hspace{1cm} z = \frac{E_2 - E_4 - E_5}{E_2}
\]
Impact on the $M_W$ measurement at the LHC

- **goal:** construct "simple" observable for $M_W^{\text{exp}}$ that makes use of both the $p_{\ell,Z}^\perp$ distribution and the precisely measured $Z$-boson mass

→ define normalized average momentum as

$$\langle p_{\ell,V}^\perp \theta[p_{\ell,V}^\perp - p_{\text{cut}}^\perp] \rangle = \frac{\int \theta^{\text{cut}} p_{\ell,V}^\perp \times \frac{d\sigma_V}{dp_{\ell,V}^\perp} \, dp_{\ell,V}^\perp}{\int \theta^{\text{cut}} \, d\sigma_V} = M_V \times f\left(\frac{p_{\text{cut}}^\perp}{M_V}\right)$$

→ define observable $M_W^{\text{exp}}$ as

$$M_W^{\text{exp}} = \frac{\langle p_{\ell,W}^\perp \rangle^{\text{exp}}}{\langle p_{\ell,Z}^\perp \rangle^{\text{exp}}} M_Z C_{\text{th}}, \quad C_{\text{th}} = \frac{M_W}{M_Z} \frac{\langle p_{\ell,Z}^\perp \rangle^{\text{th}}}{\langle p_{\ell,W}^\perp \rangle^{\text{th}}}$$
Impact on the $M_W$ measurement at the LHC

• updated theoretical description shifts extracted value of $M_W$

$$\delta M_W^{\text{exp}} = \left[ \frac{\delta \langle p_{\ell,Z}^\perp \rangle^{\text{th}}}{\langle p_{\ell,Z}^\perp \rangle^{\text{th}}} - \frac{\delta \langle p_{\ell,W}^\perp \rangle^{\text{th}}}{\langle p_{\ell,W}^\perp \rangle^{\text{th}}} \right] M_W^{\text{exp}}$$

<table>
<thead>
<tr>
<th></th>
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</tr>
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<tbody>
<tr>
<td>NLO EW</td>
<td>$1 \text{ MeV}$</td>
<td>$3 \text{ MeV}$</td>
<td>$-3 \text{ MeV}$</td>
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<tr>
<td>NNLO QCD-EW</td>
<td>$-7 \text{ MeV}$</td>
<td>$-17 \text{ MeV}$</td>
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$$\delta \langle p_{\ell,W}^\perp \rangle^{\text{th}} / \langle p_{\ell,W}^\perp \rangle^{\text{th}} \times M_W^{\text{exp}} \sim \mathcal{O}(30 - 50) \text{ MeV}$$

⇒ large cancellation by one order of magnitude

⇒ sensitive to cuts
Impact on the $M_W$ measurement at the LHC

• updated theoretical description shifts extracted value of $M_W$

\[
\delta M_W^{\text{exp}} = \left[ \frac{\delta \langle p_{\ell,Z}^\perp \rangle^{\text{th}}}{\langle p_{\ell,Z}^\perp \rangle^{\text{th}}} - \frac{\delta \langle p_{\ell,W}^\perp \rangle^{\text{th}}}{\langle p_{\ell,W}^\perp \rangle^{\text{th}}} \right] M_W^{\text{exp}}
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ATLAS $p_{\ell,V}^\perp$-cuts:
\[ \rightarrow p_{\text{cut},Z}^\perp = 25 \text{ GeV} \]
\[ \rightarrow p_{\text{cut},W}^\perp = 30 \text{ GeV} \]

⇒ moves average momentum in decays of the lighter $W$ boson towards higher values
Impact on the $M_W$ measurement at the LHC

- updated theoretical description shifts extracted value of $M_W$

$$\delta M_W^{\text{exp}} = \left[ \frac{\delta \langle p_{\ell,Z}^\perp \rangle^\text{th}}{\langle p_{\ell,Z}^\perp \rangle^\text{th}} - \frac{\delta \langle p_{\ell,W}^\perp \rangle^\text{th}}{\langle p_{\ell,W}^\perp \rangle^\text{th}} \right] M_W^{\text{exp}}$$

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find tuned value for $p_{\text{cut, } W}^\perp$ such that $C_{\text{th}}^{\text{LO}} = 1$ with $p_{\text{cut, } Z}^\perp = 25$ GeV

$$\Rightarrow p_{\text{cut, } W}^\perp = 25.44 \text{ GeV} \left(\text{instead of } p_{\text{cut, } W}^\perp = 30 \text{ GeV}\right)$$
Conclusion

• fully-differential description of QCD-EW corrections to on-shell vector-boson production
  – regularisation of infrared singularities
  – analytic computation of integrated subtraction terms
  – (on-shell form factor $pp \rightarrow W$, including two-loop master integrals with two internal masses)

⇒ QCD-EW corrections were estimated
to shift the measurement of $M_W$ by up to $-17$ MeV

Outlook

⇒ this result warrants further studies,
which incorporate all relevant details of experimental analyses
Backup
# Breakdown LHCb uncertainties

<table>
<thead>
<tr>
<th>Source</th>
<th>Size [MeV]</th>
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<tr>
<td>Parton distribution functions</td>
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<tr>
<td>Transverse momentum model</td>
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<td>QED FSR model</td>
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<td>Additional electroweak corrections</td>
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<td>Momentum scale and resolution modelling</td>
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<td>Muon ID, trigger and tracking efficiency</td>
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<td>Isolation efficiency</td>
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<tr>
<td>Total</td>
<td>32</td>
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</table>

[LHCb ’21]
Breakdown CDF uncertainties

Table 2. Uncertainties on the combined $M_W$ result.

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<tr>
<th>Source</th>
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<td>Lepton energy resolution</td>
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<td>Recoil energy scale</td>
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<td>Lepton removal</td>
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<td>Backgrounds</td>
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<td>$p_T^Z$ model</td>
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<td>$p_T^W/p_T^Z$ model</td>
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<td>Parton distributions</td>
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