Event-by-event analysis of the two-particle source function in heavy-ion collisions with EPOS

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Correspondence: kincses@ttk.elte.hu

DÁNIEL KINCSES
IN COLLABORATION WITH
M. STEFANIAK, M. CSANÁD
The HBT effect and the idea of femtoscopy

1. **R. Hanbury Brown & R. Q. Twiss (Radio-astronomy):**
   - Intensity corr. vs detector dist. ⇒ source size

2. **Goldhaber et al: analogy in high energy physics**
   - Distant star ↔ Quark-Gluon Plasma
   - Light ↔ particles from freeze-out
   - Intensity corr. of light ↔ Momentum corr. of identical (bosonic) particles
   - **Measuring source shape on the fm scale!**

\[
C(Q) = 1 + |\tilde{S}(Q)|^2
\]
Basic definitions of femtoscopical correlation functions

- **Single particle distribution:** \( N_1(p) = \int dx S(x, p) \)
- **Pair momentum distr.:** \( N_2(p_1, p_2) = \int dx_1 dx_2 S(x_1, p_1) S(x_2, p_2) |\psi(x_1, x_2)|^2 \)
- **Correlation function:** \( C(p_1, p_2) = \frac{N_2(p_1, p_2)}{N_1(p_1) N_2(p_2)} \)
- **Pair source/spatial correlation:** \( D(r, K) = \int d^4 \rho S \left( \rho + \frac{r}{2}, K \right) S \left( \rho - \frac{r}{2}, K \right) \)

\[ C(Q, K) = \frac{\int D(r, K) |\psi_Q(r)|^2 dr}{\int D(r, K) dr} \]

- **Experiments:** measuring \( C(Q) \) to gain information about \( D(r) \)
The two-particle source function (spatial correlations)

\[ D(r, K) = \int d^4 \rho S \left( \rho + \frac{r}{2}, K \right) S \left( \rho - \frac{r}{2}, K \right) \]

- **Experiments** – no direct access to pair-source
  - Assumption on the shape of the \( D(r) \) pair-source function
  - Proper description of FSI in \( \psi_Q(r) \) symmetrized pair wave function
  - Calculating \( C(Q) \), then testing the assumption on experimental data
  - Experimental indications – power-law tail for pions, Lévy-type sources?

- **Event generator models (like EPOS)** – direct access to pair-source!
  - Phenomenological investigations of \( D(r) \) possible
What is the shape of the source? Gaussian vs. Lévy distributions in heavy ion physics

\[ S(r, K) = \mathcal{L}(\alpha(K), R(K); r) = \frac{1}{(2\pi)^3} \int d^3 q e^{iqr} e^{-\frac{1}{2}|qR|^\alpha} \]

**Possible (competing) reasons for the appearance of Lévy-type sources:**
1. Proximity of the critical endpoint
2. Anomalous diffusion
3. Jet fragmentation
4. Event averaging (different shapes)?

**Symmetric Lévy-stable distribution:**
- From generalized central limit theorem, power-law tail (if \( \alpha < 2 \)) \( \sim r^{-(1+\alpha)} \)
- \( \alpha = 2 \) Gaussian, \( \alpha = 1 \) Cauchy
- Retains the same \( \alpha \) under convolution

\[ S(r) = \mathcal{L}(\alpha, R; r) \Rightarrow D(r) = \mathcal{L}(\alpha, 2^{1/\alpha} R; r) \]

Csörgő, Hegyi, Novák, Zajc, AIP Conf.Proc. 828
Metzler, Klafter, Physics Reports 339 (2000) 1-77,
The EPOS model

• Energy conserving quantum-mechanical multiple scattering approach, based on Partons (parton ladders), Off-shell remnants, and Splitting of parton ladders.

• The model is based on Monte-Carlo techniques
• Theoretical framework: parton-based Gribov-Regge theory (PBGRT)
• Three main parts of the model:
  • Core-Corona division (based on dE/dx of string segments)
  • Hydrodynamical evolution (vHLLE 3D+1 viscous hydro)
  • Hadronic cascades (UrQMD afterburner)
Details of the analysis

• \( \sqrt{s_{NN}} = 200 \text{ GeV} \) Au+Au collisions generated by EPOS359

• Observable:
  angle-avg. radial source distribution of like-sign pion pairs
  \[
  D(r_{1,2}^{L_{CMS}}) = \int d\Omega dt D(r)
  \]

• Investigated cases:
  1. CORE, primordial pions – Gaussian source shape*
  2. CORE, decay products incl. – power-law structures appear*
  3. CORE+CORONA+UrQMD, primordial pions – Lévy-shape*
  4. CORE+CORONA+UrQMD, decay products incl. – Lévy-shape

\[
\begin{align*}
  r_{1,2}^{L_{CMS}} &= \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z_{L_{CMS}})^2}; \\
  \Delta z_{L_{CMS}} &= \Delta z - \frac{\beta(\Delta t)}{\sqrt{1 - \beta^2}}; \\
  \beta &= \frac{p_{z,1} + p_{z,2}}{E_1 + E_2}
\end{align*}
\]

*for details see backup slide!
Details of the analysis

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*for details see backup slide!
Example single evt. fit – CORE+CORONA+UrQMD with primordial + decay pions

- Investigating $D(r)$ event-by-event
- Lévy-fits provide good description (2-100 fm range)
- Let’s repeat such fits for thousands of events
- Extract $\alpha, R$ distribution
Example $\alpha, R$ distribution – CORE+CORONA+UrQMD with primordial + decay pions

- Normal distr. of $\alpha, R$ for given centrality & kT
- Extract mean and std. dev, investigate centrality and mT dependence
- kT dependence investigated around the peak of the pair-kT distr. to have adequate stat.
\( \langle \alpha \rangle, \langle R \rangle \text{ vs. } m_T, \text{ centr.} \)

CORE+CORONA+UrQMD primordial + decay pions

- Trends, magnitudes of \( R \) similar to experimental results
- Higher magnitudes of \( \alpha \) than experimental results

\( R \sim 8.5-7 \text{ fm} \)

PHENIX 0-30\% Au+Au @200 GeV

PHYS.REV. C97 (2018) no.6, 064911
\[\langle \alpha \rangle, \langle R \rangle \text{ vs. } m_T, \text{ centr.} \]

CORE+CORONA+UrQMD primordial + decay pions

- Trends, magnitudes of R similar to experimental results
- Higher magnitudes of \( \alpha \) than experimental results

\[\alpha \sim 1.4-1.2 \]

PHENIX 0-30% Au+Au@200 GeV

\[\alpha_0 = 1.207, \chi^2 / NDF = 208/61, \text{ CL} < 0.1\% \]

\[\text{PHENIX 0-30\% Au+Au@200 GeV} \]

\[\text{Phys.Rev. C97 (2018) no.6, 064911}\]
Summary – event by event analysis of the pion pair-source in EPOS 200 GeV Au+Au collisions

1. Single event Levy fits to angle-averaged $D(r)$ – event-by-event non-Gaussianity

2. Extracting the mean, std.dev. of $R$, $\alpha$ distr.

3. Investigating mT & centr. dependence
   - Lévy fits provide good descr., power-law tail strongly affected by rescattering, decays
Outlook

Investigating the pair-source in multiple dimensions

Investigating the pair-source of different particles (kaons, protons)

Reconstruct correlation func. from measured pair-source

If you are interested in similar topics come to the Zimányi Winter School!
http://zimanyischool.kfki.hu/22/

See details in Entropy 24 (2022) 3, 308
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Thank you for your attention!
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INTERNATIONAL SCHOOL OF SUBNUCLEAR PHYSICS, ERICE, 2022
Backup

CORE only: Gaussian source shape!

CORE + decays: already power-law structures!
• Removing decay pions decreases R, increases $\alpha$
  (but still far from Gaussian)

• Decays and rescattering both play an important role in the appearance of the power-law behavior

see also other phenomenological studies e.g. Universe 5, 148, Phys. Part. Nucl. 51(3), 282–287

$\langle \alpha \rangle, \langle R \rangle$ vs. $m_T$, centr. CORE+CORONA+UrQMD
decay pions excluded(!)
Backup

- Event-averaged source not perfectly Lévy
- Nevertheless, very similar parameters
  - Event averaged: \( \alpha \approx 1.62, R \approx 9.15 \text{ fm} \)
  - Event-by-event: \( \alpha \approx 1.66, R \approx 8.96 \text{ fm} \)
- More reasonable approach for kaons
Backup - HBT and the phase transition

- \( C(q) \) usually measured in the Bertsch-Pratt pair coordinate-system
  - **out**: direction of the average transverse momentum
  - **long**: beam direction
  - **side**: orthogonal to the latter two

- \( R_{\text{out}}, R_{\text{side}}, R_{\text{long}} \): HBT radii

- \( \Delta \tau \) emission duration, i.e. \( S(r, \tau) \sim e^{-\frac{(\tau-\tau_0)^2}{2\Delta \tau^2}} \)

- From a simple hydro calculation:
  \[
  R_{\text{out}}^2 = \frac{R^2}{1 + u_T^2 m_T/T_0} + \beta_T^2 \Delta \tau^2, \quad R_{\text{side}}^2 = \frac{R^2}{1 + u_T^2 m_T/T_0}
  \]

- RHIC, 200 GeV: \( R_{\text{out}} \approx R_{\text{side}} \rightarrow \) no strong 1st order phase trans.

- Plus lots of other details: pre-equilibrium flow, initial state, EoS, ...

Backup – 2nd order phase transition?

- Second order phase transitions: critical exponents
  - **Near the critical point**
    - Specific heat $\sim (T - T_c)/T_c^{-\alpha}$
    - Order parameter $\sim (T - T_c)/T_c^{-\beta}$
    - Susceptibility/compressibility $\sim (T - T_c)/T_c^{-\gamma}$
    - Correlation length $\sim (T - T_c)/T_c^{-\nu}$
  - **At the critical point**
    - Order parameter $\sim (\text{source field})^{1/\delta}$
    - Spatial correlation function $\sim r^{-d+2-\eta}$
    - Ginzburg-Landau: $\alpha = 0, \beta = 0.5, \gamma = 1, \eta = 0.5, \delta = 3, \eta = 0$
  - QCD $\leftrightarrow$ 3D Ising model
  - Can we measure the $\eta$ power-law exponent?
  - Depends on spatial distribution: measurable with femtoscopy!
  - **What distribution has a power-law exponent?** Levy-stable distribution!
Backup –
Properties of univariate stable distributions

- **Univariate stable distribution:**
  \[ f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \varphi(q)e^{-ixq}dq, \]

  where the characteristic function:
  \[ \varphi(q; \alpha, \beta, R, \mu) = \exp(iq\mu - |qR|^\alpha(1 - i\beta\text{sgn}(q)\Phi)) \]

- \( \alpha \): index of stability
- \( \beta \): skewness, symmetric if \( \beta = 0 \)
- \( R \): scale parameter
- \( \mu \): location, equals the median,
  if \( \alpha > 1 \): \( \mu = \text{mean} \)

- **Important characteristics of stable distributions:**
  - The distributions retain the same \( \alpha \) and \( \beta \) under convolution of random variables
  - Any moment greater than \( \alpha \) isn’t defined

\[ \Phi = \begin{cases} \tan \left( \frac{\pi\alpha}{2} \right), & \alpha \neq 1 \\ -\frac{2}{\pi} \log|q|, & \alpha = 1 \end{cases} \]
Backup - Lévy index as a critical exponent?

- Critical spatial correlation: $\sim r^{-(d-2+\eta)}$;
  Lévy source: $\sim r^{-(1+\alpha)}$; $\alpha \Leftrightarrow \eta$?
  

- QCD universality class $\leftrightarrow$ 3D Ising
  

- At the critical point:
  - Random field 3D Ising: $\eta = 0.50 \pm 0.05$
  - 3D Ising: $\eta = 0.03631(3)$

- Motivation for precise Lévy HBT!
- Change in $\alpha_{\text{Lévy}}$ - proximity of CEP?

- Modulo finite size/time and non-equilibrium effects
- Other possible reasons for Lévy distr.: anomalous diffusion, QCD jets, …